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ABSTRACT

This booklet describes the Elementary Mathematics through Teacher Partnerships (EMTTP) project in which the faculty at a K-6 elementary school and a few mathematicians and mathematics educators met regularly to talk about mathematics and teaching. This book illustrates how the partnership grew and learning occurred as a result. The difficulties that were encountered, the decisions that were made, and the benefits derived from the efforts to help others who may be involved in similar work are described. It also examines how a conversation about mathematics and teaching grew as mathematicians and elementary school teachers worked together in the elementary classroom. (Contains 34 references.) (ASK)

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PARTNERS IN TEACHING MATH

School and University Faculty Examine Issues in Mathematics Education

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by
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and
Herb Clemens, Diane Hunter, Carol Lubomudrov
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PREFACE

Mathematics education today is faced with many questions that do not seem to be readily answered by simply developing new goals, or designing new curriculum. There is a growing recognition that improving mathematics education at all levels will require a collaboration between school and university teachers that pays closer attention to classroom practice and to connections between mathematical content and pedagogical knowledge. However, there are few models for such a collaboration. In the past when these groups have joined forces, the expectations of both have been that the university faculty will direct the project and school faculty will implement what has been developed in the classrooms. This model has not proved to be very effective.

This book describes a project that took another approach. In the *Elementary Mathematics Through Teacher Partnerships* (EMTTP) project, the faculty at a K-6 elementary school and a few mathematicians and mathematics educators met regularly to talk about mathematics and about teaching. Our goal was to foster an extended conversation between school and university faculty based on the mathematics of the elementary school, one in which each group would contribute to the discussion and assume some responsibility for the direction of the work. During the two years of the project, a rich dialogue developed between those participating in the project. As the name of the project and the title of this book suggest, this dialogue evolved into a partnership across the grades that is especially important given the current changes in mathematics education. My purpose in writing this book is to illustrate through the events of the project how our partnership grew and what we learned. By describing the difficulties we encountered, the decisions we made, and the benefits we derived from our efforts, my hope is that our experiences will provide useful information for others who may be involved in similar work.

That this partnership had an opportunity to grow at all is due to the efforts of Herb Clemens, the project director and of Carol Lubomudrov, the principal at Washington Elementary. Without Herb's dedication to mathematics education and his persistence in developing the project, without Carol's leadership and commitment to her faculty's professional growth, the EMTTP project would not have taken place. Those of us who were a part of the project owe them our thanks for their vision and their support.

In addition to Herb and Carol, several others made significant contributions to the EMTTP project. Deborah Ball, Michigan State University, through her demonstration lessons provided us with an invaluable context in which to reflect on teaching mathematics. Deborah Schifter, Educational Development Center, Boston, gave us the benefit of her years of experience in projects in mathematics education and through her evaluations of the project, helped us to determine new directions for our work. Lorel Preston, a graduate student in mathematics education, spent many hours working with me. She visited classrooms, attended meetings, and throughout the project offered many thoughtful suggestions that contributed enormously to our efforts.

However, the major acknowledgement for the success of the EMTTP project must be given to the school faculty who participated in the work. It was they who shared their classroom work with us, who helped us understand issues about student learning that we had overlooked. And, it was they who walked along a path in mathematics, one that sometimes seemed uninviting, because we asked them to come and look at what seemed so valuable to us. Without their courage and support our partnership would have been an empty exercise. I want to thank these teachers not only for the pleasure of the work we shared, but also for their willingness to let me use their papers, reports, our conversations and our experiences to help describe the work of the project. (Except where they have asked me to use their own name, I have used pseudonyms throughout the book in referring to a specific teacher or student.)

Many people have contributed to the writing of this book. In particular I want to thank my co-authors. Diane Hunter, Carol Randell, Scott Smith, and Lorel Preston have described their own work in the project in detail so that we may learn from their experiences (See Chapter 6). Herb Clemens and Carol Lubomudrov have provided us with their reflections on the project and its position in the school/university community (See Chapter 7). Both Deborah Schifter and Herb Clemens have helped me to refine the direction of the manuscript with their questions and comments at each stage in the writing process. Throughout the project, Patricia Erdmann and Jennifer Johnson have shared many discussions with me about the project events, elementary mathematics, and my course for preservice teachers. Not only were the insights they offered and their careful reviews of my drafts greatly appreciated, but their support and encouragement were essential.

Finally, I must thank my husband, Paul, for his confidence in me, and my son Paul Leo who said, "Mum, don't worry about it- just write it."

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CHAPTER 1

INTRODUCTION

This chapter presents an overview of the EMTTP project, describing the background of the project, of the participants, and of the school where the work took place. The organization of the project work is explained and the project results are summarized briefly.

This book is about a project in mathematics education called *Elementary Mathematics through Teacher Partnerships*, or EMTTP. It describes a partnership that evolved between school and university faculty as they worked together doing mathematics and discussing ways to teach and learn mathematics. Educational reform is certainly not new, so one must ask what there is about this project that can contribute useful information to the general knowledge about mathematics education. Reform efforts in the past have generally left the question of which topics to study to the mathematicians, and the methods of teaching mathematics to the school teachers and mathematics educators. This separation has overlooked an important aspect of the nature of learning. That is, *how one learns* a subject and the image one forms of that subject area can have a large impact on *what one will learn* (or as a teacher, *what one will teach*) about the subject now and in the future (Shavelson, R.J. and Stern, P, 1981; Schifter, D. 1993). This seems especially true at the elementary school level where children are beginning to develop habits of learning and to form their first impressions of what doing mathematics involves. Today there is a growing recognition of the need to involve the three groups responsible for teaching mathematics, school teachers, mathematics educators, and mathematicians in a dialogue not only about the topics to be studied, but also about ways to teach mathematics. But what conditions, what kinds of interaction will foster a meaningful discussion among these groups? Certainly all three groups are involved in mathematics and in teaching, but their professional background, experiences, and goals in education are often very different. Can the groups come to appreciate each other's perspective? What does each group bring to the dialogue, what can they learn in the process, and how will it affect mathematics education? The project described here examined these questions while developing a rich dialogue between the faculty at a K-6 elementary school, members of a mathematics department at a state university, and researchers in mathematics education. How this dialogue evolved, and what resulted from this exchange, is the focus of this book.

The EMTTP project, funded by the National Science Foundation, began in August 1992 and ended in June 1994. Herb Clemens of the Mathematics Department at the University of Utah directed the project and he invited me, also from the Mathematics Department, to join him in organizing the project work and leading the school-year program for the teachers. Each of us had been involved in developing mathematics courses for preservice teachers and had organized summer programs in mathematics for practicing teachers. In addition, we had both worked with Carol Lubomudrov who was the principal of Washington Elementary, the school involved in the EMTTP project. For several years,

Herb and I had taught small classes of students at schools where Carol was the principal. During this time the three of us had shared our thinking about elementary school mathematics and ways to help school teachers, both preservice and inservice, teach mathematics more effectively. Knowing that many practicing K-6 teachers had only limited experience in mathematics in their own education, Carol was pleased to see that we were developing a more extensive mathematics preparation for preservice teachers. However, she saw a need for this coursework to be related to actual classroom situations and to the different ways in which young children think about mathematics. Working with students and teachers at Carol's schools was very useful to Herb and me in our own work, but Carol was hoping to find a way that this interaction with the university might benefit her school and her faculty as well.

As an elementary school principal, she believed that providing her faculty with opportunities to continue their own learning was a key factor in maintaining strong elementary school programs. When Carol became the principal at Washington Elementary two years prior to the project, she initiated a dialogue with the faculty on various approaches to student learning and together they had considered ways to improve the school's mathematics program. Like many inner-city, Title One schools, Washington Elementary, had a large minority population (nineteen different first languages being represented among the students) and a high student turnover rate during the school year. The school faculty was historically a very interactive group of teachers, used to dealing with the many social problems of their students. As in many elementary schools today, the mathematics curriculum focused primarily on arithmetic presented as a series of rules for computation, and students were grouped in math classes by their ability in computation. Now with Carol's enthusiasm and support, the faculty evinced a willingness to examine the process of teaching and learning in the context of their mathematics program and to take part in an extended professional development effort.

What shape should this work take? What role would each group, university and school faculty, play? From our own work in elementary schools, both teaching classes and working with school faculty, Herb and I believed that improving school mathematics programs is not simply a question of changing the topics taught, or the manner of teaching mathematics. Nor is new mathematics learning for teachers sufficient. While each of these changes may have some benefit, and all are necessary to make a substantive impact on a school mathematics culture, there is another crucial consideration. Since it is teachers who bring about change in the classroom, teachers need to understand the rationale for change. Then, based on this understanding, they themselves must determine the implications for their teaching practice and, together with their colleagues, evaluate the effect of changes made. There are many examples which demonstrate that teachers' uncritical acceptance of reform ideas proposed by experts in content and/or pedagogy can lead to a mathematics instruction that runs counter to the intent of the proposed reforms (Ball, 1990; Remillard, 1992). Consulting with outside experts on content and pedagogy is important, but ultimately, how one incorporates this information into one's teaching practice is what will determine growth in teaching mathematics. This individual, reflective growth, combined with a school-wide sense of direction, seems essential for lasting improvement in

the mathematics learning environment at a school. To support this kind of professional development we believe that a partnership between school and university faculty is quite useful, one that recognizes the expertise each group brings, and is based on the willingness of those involved to consider and learn from each other's perspective, both on mathematics and on the process of teaching and learning mathematics.

This idea of a true partnership in which each group assumes some responsibility for the direction of the work is very different from the usual school/university interaction. We realized early on that, if in fact such a relationship was possible, it would take time to develop and it would require us all to take on new roles. Our goal in the EMTTP project was to foster an extended conversation between school and university faculty, one that would be based on mathematical experiences in elementary school classrooms and one to which both groups would contribute their knowledge and understanding. Each group stood to gain from this conversation: the school faculty would have the opportunity to consider, in some depth, the mathematics they taught, what and how their students were learning; and as a faculty find ways to improve the mathematics program at Washington Elementary; university faculty working with teachers in elementary school classrooms could learn to deal with complex pedagogical situations, and determine ways to improve their own teaching, especially in the mathematics courses for prospective teachers.

Working with Carol and with the teachers, Herb developed a plan for the EMTTP project that would involve the entire school faculty over a two-year period. The project was organized around two main components, an intensive three to four week summer workshop, and a follow-up program during the school year. In both our initial planning and in the changes we made along the way, we benefitted a great deal from the advice and support of two professional mathematics educators, Deborah Ball and Deborah Schifter. Deborah Ball, at Michigan State University, is a distinguished researcher in mathematics education who chaired the working group, 'Mathematics Teaching', in developing the National Council of Teachers of Mathematics (NCTM), *Professional Standards for Teaching Mathematics*. As a part of her research she teaches the daily mathematics lesson in a nearby third grade classroom. Deborah Schifter, now at the Educational Development Center in Boston, is a former director of the well-known SummerMath for Teachers Program at Mount Holyoke College in Massachusetts. As they reviewed our plans, both emphasized the importance of giving teachers the opportunity to examine not only the math concepts, but also their own thinking processes, even as they concentrated on math during the first two weeks of each summer workshop. They saw this consideration as an important step in developing new images of teaching math. To help us in this work, Deborah Ball agreed to give periodic demonstration lessons and lead the debriefing of the lessons afterwards, and Deborah Schifter agreed to serve as an internal evaluator on the project who would visit us 2-3 times a year and share with us her observations on the progress of our work.

In the remaining chapters I will describe the summer workshop and school-year program in detail, but first will outline each of these components briefly and discuss how they fit into the goal of developing a partnership in mathematics education.

Summer Workshop. Each year's summer workshop was held at Washington Elementary, beginning towards the end of July, and ending just prior to the start of the school year. During the first two weeks the workshops focused on mathematics content and pedagogy. The teachers selected one theme per week to follow in morning math sessions. The sessions began with problems and/or questions on the chosen theme, questions that would challenge the teachers mathematically, could be explored at several levels of expertise, and would relate directly to classroom work at the K-6 level. Working on these questions led to important pedagogical issues. Teachers and project staff shared ideas of how one learns mathematics, which models work best, and what the difficulties are in mastering certain concepts. Each day also included teacher presentations on solutions to the problems and connections to the classroom. The main focus of this two week period was to give teachers an opportunity to learn more about elementary mathematics, but to do this in a way that encouraged them to ask questions about concepts, think about their understanding, critique their answers, and discuss their work with their colleagues. Then, using their own experience as learners in this context, they could consider their role in their students' learning.

During the last week of each summer workshop Deborah Ball gave demonstration lessons and led debriefing sessions. For three days Ball taught one primary (grades1-3) and one intermediate (grades 4-6) class, using students who attended Washington Elementary, with the teachers as observers. Briefing sessions were held before and after the classes. The teachers could discuss with Ball what worked, what didn't work, and what direction the next session might take. This activity set the stage for a collegial analysis of instructional decisions and their effectiveness, explorations of possible alternatives and of the children's thinking and evidence of their understanding of concepts. This group discussion was to be a model for faculty exchange during the year, an exchange to be centered on evaluating lessons and modifying instruction based on mathematics goals and student response.

School Year Support. Making significant change in teaching requires the time to think seriously about one's efforts and their impact in the classroom. The examples in Deborah Schifter's book, *Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenge of Reform* (Schifter and Fosnot, 1993), made it very clear that the process of examining one's ideas about teaching math requires time and a great deal of effort. However, the typical elementary teaching schedule affords little time for reflection, or opportunity to discuss one's efforts with colleagues. Support and feedback during the school year are critical if teachers are to try out ideas from a summer workshop. Many teachers will attest to the fact that summer workshop plans are often placed on the top shelf and not used once the more immediate concerns of the classroom arise. This is particularly true in an inner-city school where the many non-academic problems of one's students can be so demanding. Moreover, when teachers begin to change their mathematics practice, they will encounter new problems and concerns. Opportunities to continue learning about mathematics and to discuss one's teaching with colleagues can be especially important in supporting a teacher's efforts to modify their teaching and to evaluate the changes they are making.

Teacher-participants in most professional development programs are drawn from a number of different schools, so that when the teachers return to their classrooms for the school year, there may be only one or two colleagues at their school with whom they can share their work and discuss their goals. Because the EMTTP project involved virtually the entire school faculty, we had a natural setting for the kind of faculty interaction necessary to improve a mathematics program school-wide. To continue the conversation about mathematics begun in the summer, I met biweekly during the school year with the teachers at each grade level and worked with teachers individually in their classrooms. A graduate student in mathematics education, Lorel Preston, assisted me in this work. These regular, forty-five minute, grade level meetings gave teachers an opportunity to share their math lessons, refine their thinking on both mathematics content and pedagogical issues, and consider long range goals for their mathematics curriculum. By working in the classrooms with the teachers, giving lessons or leading groups of students ourselves, and debriefing afterwards, we were able to learn about an individual teacher's goals and concerns. Sharing the elementary teaching experience in this way, we were better able to offer support that recognized each teacher's potential for growth and could relate the actual classroom work to the grade level discussions.

The EMTTP project is now over, but the conversation about teaching mathematics begun during the project still continues. Is this conversation beneficial, did school and university faculty form an effective partnership, and if so, what was learned in the process? The answers to these questions are best illustrated by the reader's critical examination of the events described in the remaining chapters. However, we are encouraged by several developments. Teachers report benefits to themselves and their students from their work in the project. As a faculty they believe in the value of their grade level meetings. In the two years since the project ended, they have continued these meetings with school funds paying for release time. In addition, they have organized three or four math sessions during each school year at which project staff led discussions on topics in mathematics education. Several of the school faculty have continued to participate in mathematics workshops and several belong to a network of teachers who meet twice a month to examine topics in mathematics education and communicate electronically with teachers across the country (This network, the *Teachers Resource Network*, is funded by the DeWitt Wallace–Reader's Digest Fund and directed by EDC's Center for Development of Teaching.). Three of the particpants in the project are pursuing or have completed graduate degrees in education. One of these three is now in the second year of teaching a university math methods course for early childhood teaching majors.

The teachers report that the project work has affected their students' towards mathematics. Now students see mathematics as both more accessible and more related to their daily experience. Students are more willing to take risks and to tackle complex problems. The teachers have seen the students' confidence grow as they have come to realize that doing math involves thinking and reasoning about problems as well as learning facts or rules. The teachers find that students have grown in their ability to explain their thinking in mathematics and that this process of discussion has carried over into other areas as well. It is natural to consider whether these improvements in the students' mathematical work

reported by the teachers can be measured in some objective way such as with increased scores on standardized tests. While it is unclear whether SAT scores can measure the strength of a mathematics program or the kind of long-range learning goals of the EMTT Project, we have tracked those students who were at Washington and took the SAT in the years 1992, 1993, 1994 (at the beginning of the project, half-way through the project and just after the project ended). With the high student turnover rate at the school this group consists of 68 students from three different classes (students who were in the 4th, 5th, and 6th grades respectively at the end of the project or in the 1994-95 school year). Overall, the total math scores remained relatively stable even though the teachers have stopped "teaching to the test" and have emphasized conceptual understanding and mathematical dialogue. When we look at these three classes separately, the 5th and the 6th grade median math scores show an increasing trend, but the 4th grade median scores declined not only in math, but also in reading. In hindsight we regret not having attempted an alternative method of assessment of students' progress in the conceptual and reasoning areas on which the project focused. However, the national discussion around developing such methods indicate that this is not an easy task and that there may not be clear answers on which to base comparisons of schools or to evaluate the impact of efforts such as the EMTTP project on student achievement.

Through our own work in this project we at the university have come to understand better the complexity of the elementary school classroom. The issues faced by elementary school teachers on a daily basis go far beyond issues of mathematics content. We cannot hope to succeed in dealing with questions of content, mathematics or otherwise, unless we examine these issues within the context of classroom culture, and student and teacher interaction. Working in elementary classrooms with practicing teachers, we've learned new ways to approach our own preservice mathematics classes. Our university courses still focus on helping preservice teachers develop a conceptual framework for the K-6 mathematics curriculum. However, by comparing various mathematical models with the teachers, examining student work, and critiquing lessons that we've given, we have developed better ways to connect our course concepts at the university with the pedagogical issues that our students will face. This connection will enrich our students' understanding both of mathematics and of teaching. We believe even more strongly in the need to provide preservice teachers with a mathematics program that integrates both content and pedagogical knowledge and is classroom-based. It is not realistic to imagine that this will take place unless closer connections are established between the different groups of university faculty working in mathematics education, specifically those in content and those in pedagogical areas. Currently we are working with colleagues in the Educational Studies Department to better align mathematics and mathematics methods courses and to develop close ties to practicing teachers in school classrooms. Collaborations such as the EMTTP project, which establish connections between those involved in teaching mathematics at all levels, seem an important step in this process and an essential part of improving both precollege and college mathematics programs.

CHAPTER 2

BEGINNING A CONVERSATION

The EMTTP project began with an intensive four week summer workshop in August 1992. This period was the first stage in a conversation about mathematics and about teaching that developed gradually between school and university faculty over the two years of the project. How the discussion began and what issues were raised as work on mathematical problems led to pedagogical questions are the topics examined in this chapter. (August 1992)

It was the last week in July 1992 when we all met in the Washington School library to begin our work together on the EMTTP project. We were expecting about forty participants, mostly from the Washington faculty and staff. There were to be ten teachers coming from other schools in the Salt Lake City District, as well as two parents of Washington School students. The round tables in the library filled up quickly as teachers met their colleagues just back from summer vacation and filled out the information forms in their registration packets. Carol began the meeting promptly at 8:30 am by welcoming everyone to Washington School and then introducing Herb and me to the group. Letters had been sent to all of the participants describing the four week summer workshop that was just beginning, but Herb summarized the plan again briefly, and then asked for questions. During the first two weeks, the participants were to select one topic each week that they would follow in morning math sessions. There were four topics to choose from, Fractions, Number Systems, Estimation, and Geometry. To help them decide which to choose, the teachers asked us to say a few words about what we would do with each topic. As Herb and I took turns outlining what we hoped to do and the teachers considered which topics they would select, I thought back over our plans for these math sessions and worried how the teachers might react. Did our plans really make sense?

Both in planning the summer math work and in the letters to the teachers about the summer program, we had used the word *session* rather than *class*. Herb and I did not see the workshop as a set of classes to be taken, but more as mini-seminars or discussions in which all of us participating in the workshop would need to contribute our ideas. We were planning to begin each day by presenting some questions or problems on the topic chosen. After working on the mathematics, we hoped to have a discussion in which we would all share our thinking about the concepts involved and relate this work to the elementary mathematics curriculum. I was leading the sessions on Number Systems and Estimation and I had spent a long time thinking about which topics to examine. I wanted to offer some questions that would challenge what for many teachers is a procedural picture of dealing with numbers. Would looking at the Base 5 number system be a good way to examine the structure of a number system and the meanings of the arithmetic operations? Working in an unfamiliar number system can help one consider some very good mathematical concepts about how number systems in general work.It can also be, at least initially, very frustrating! Would we be able to get past that point in a week's time, and would it lead to a conversation

about classroom connections? At this point, I wasn't sure! One can plan a course, but a conversation needs to develop and the kind we hoped to take part in can't simply be mandated. For a real exchange of ideas to take place, those involved need to have some understanding of each others' perspective and, in addition, be confident that their thinking will be valued, even if their point of view is not shared by all in the group. Certainly this would take time, but given our different backgrounds and interests, was it even realistic to think that such a dialogue was possible? Even if a conversation did develop, would it be mutually beneficial, or were our goals too different?

While the teachers as a faculty were used to meeting to examine school issues, there tended to be very little discussion about curriculum and broad educational goals. How many of the teachers would be willing to sit down and consider this now, in a subject area with which many people are uncomfortable, and with supposed experts in that subject? Surely it would be much easier to come expecting to be a student and let the experts be responsible for the discussion as is often the case with in-service programs in mathematics. In many ways this was a tempting solution for me too! I would feel more comfortable focusing on content questions rather than trying to deal with issues of pedagogy. Although I was a teacher too, my own experience in teaching elementary mathematics was limited at best. Yet our goal was precisely to change the model where content and pedagogy are treated separately, a model in which university faculty are expected to play a directive role. The latter model has been tried and found ineffective, leading to problems in communication between "experts" and participants and resulting in precious little carryover of project work into classroom instruction. Providing opportunities for the teachers to learn more about mathematics was a primary concern, but to make this effective, we "content experts" ourselves had a lot to learn about elementary math pedagogy from the teachers' classroom experience! It was the blending of the two perspectives on teaching mathematics that seemed to us to be the key to developing a strong mathematics program.

After our brief descriptions of the sessions we'd planned, our first meeting ended and we headed off to the classrooms to meet with each of our groups of teacher-participants. In making their selections the teachers choose to leave the topic of Fractions till the second week and made their selections of two topics for the first week's study from Number Systems, Geometry, and Estimation. This meant that we had three groups each of which was fairly well-balanced in size and had a range of teachers from kindergarten through the sixth grade levels. As we began I felt a lot of uncertainty about how the workshop would proceed, and it was clear that we would all need to be very flexible as we learned to work together. But it was good finally to be underway after all of the planning during the last few months! Carol, Herb, and I were looking forward to working together, and the fact that virtually the entire school faculty at Washington Elementary was involved presented an exciting challenge.

Working on Math Together

During the first week of the workshop we could see that the math sessions we'd

planned were quite different from what the school faculty had anticipated, despite our earlier discussions about the workshop. Many participants had come expecting that we would present some methods and materials to use in teaching a math topic and that they would practice these new techniques. They were not prepared for the extent to which we intended working on math problems. The amount of time we spent on a single problem and the lack of direction we offered in finding solutions to problems surprised the teachers. Nevertheless, questions such as, "Why does the 'invert and multiply' rule work in division of fractions?", "How can you tell which numbers are even in Base 5?", and "How much is the $\sqrt{3}$ actually?" were shaking some assumptions about how numbers worked. This was an exhilarating experience for some participants as old ideas were challenged, and they began to form new images of doing mathematics. Later on in writing about her experiences during the math sessions, one teacher described this feeling with the words,

> "I had to think harder than I have in a long time – I appreciated the challenge and know this is what we need to do with our own students".

Another teacher wrote,

> "it opened up a new world of 'I never thought of it that way' ",

and Carol reported to us delightedly that she heard her faculty "having math arguments" in the halls. However, it was clear to us that not all of the teachers shared these feelings. Some participants did not see the topics we considered sufficiently connected to their classroom work, and others felt that the open-ended nature of the math work was too demanding. They had little sense of how to proceed or to take part in the discussions. And, a number would have agreed with the following comments taken from the exit surveys,

> "Too complicated; too open-ended; would have liked it on a different math level."

By the end of the first week, despite some concerns on both sides about how best to work together, we had become a bit more comfortable with each other. Our daily work had developed into a familiar pattern. The morning was divided into four forty-five minute periods with a five minute break between periods. Those following a particular theme for the week, such as Number Systems, would meet for three periods to consider that topic. During the fourth period they could use the computer lab, watch math videos, or examine the math resources in the library. In the first period I would begin with some questions about the topic, which we would consider as a group. Then I would leave the teachers themselves to examine these or related problems further, in the second period. While I was away, Jennifer Johnson, a colleague from the Mathematics Department was available for questions from the teachers. In the third period, the teachers would present the results of their work and we would consider how this topic related to their classroom work.

One morning during the second week as I walked back to meet the Number Systems group for this third period, I wondered what connections the teachers had made between

the concept of division and that of fractions. The list of questions I'd left the group to consider at the end of the first period included: "Is $3/4 = 3 \div 4$. Explain your answer." As I went to another room to talk about estimation with a different group of teachers I heard comments like,

> "Well, of course $3/4 = 3 \div 4$!", "Okay then, can you show me why?", and " Is this really supposed to be a question?".

It is initially hard to understand what is meant by a 'concept' of division if one sees this operation as simply a procedure to be carried out! Knowing that $28 \div 4$ equals 7 is important, but is quite different from understanding the roles that 4 and 7 play and that these roles can vary in different problem situations. It is the latter kind of understanding that offers the kind of knowledge on which to base a picture of the operation itself, and to relate this operation to other concepts.

As I entered the room, I saw that the teachers were still considering the question, "Is $3/4 = 3 \div 4$? Explain your answer." One group suggested that they were the same because they both equalled .75 on a calculator when you used the division key for $3 \div 4$, and converted the fraction 3/4 to a decimal using the fraction-to-decimal key. Another teacher used the paper and pencil division procedure to show $3 \div 4 = .75$, and then said that since 3/4 and 75/100 are equivalent fractions, $3/4 = 3 \div 4$. Others in the group felt uneasy about using the calculator as an authority in the situation and several felt that there should be some *nice* way to show why these symbols represented the same amount. Two other teachers thought they had a way to do this. One of them drew a rectangular bar on the board, divided this bar into four equal sections, and then shaded in three of the sections, saying this was a picture of 3/4. Their diagram is shown below in Figure 2.1.

Figure 2.1

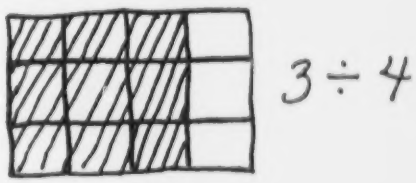


Figure 2.2

Now, to show $3 \div 4$, the teacher working at the board expanded her original drawing to form the diagram in Figure 2.2 above. As she tried to explain why this showed $3 \div 4$ to the group, it became clear that while she could point to parts of the picture that showed the numbers 3, and 4, the diagram did not seem to illustrate the division process she'd intended. Teachers in the group saw this as another picture of 3/4 instead. As the group continued to discuss the diagrams, it became clear that it was easy to draw a diagram representing 3/4, the most common one drawn being a circle divided into four equal sectors, with three of the sectors shaded. Drawing a picture to represent $3 \div 4$ caused

more of a problem for these teachers who generally taught the first through third grades and were more accustomed to looking at problems of the form $a \div b$, where $a > b$.

Valerie, one of the teachers in the group, had been working with Jennifer. From their discussions, Valerie had began to look at what $3 \div 4$ might mean and at this point she volunteered her thinking and presented the diagram below in Figure 2.3. She said she would begin as the children would, by drawing three circles or pizzas. Then she divided each pizza in half to see if that would come out evenly in sharing the pizza with four children. Since it didn't, she proceeded to divide each half, again in half. This gave her a total of 12 equal pieces. Now, each child could have three slices of pizza. Although the rest of the teachers followed Valerie's procedure, there was some uncertainty about why this diagram showed $3/4 = 3 \div 4$. So, Valerie continued her explanation by saying that just as $6 \div 3$ could be looked at as 6 cookies shared equally among 3 children, $3 \div 4$ means how much pizza each of the four children would get when they share three pizzas equally. Because in her diagram, each child got three slices, or three-fourths of a pizza, this shows that $3/4 = 3 \div 4$.

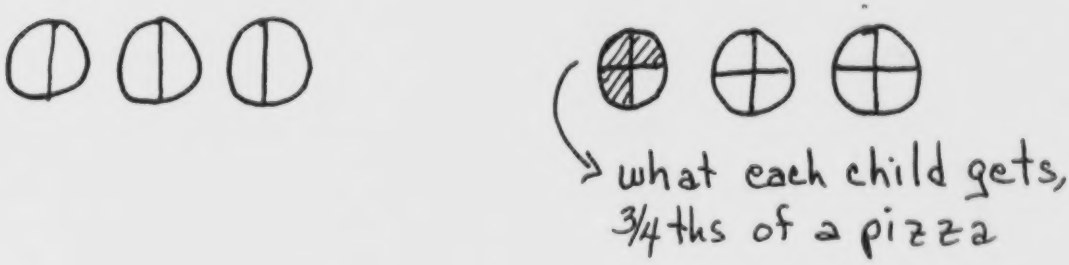


Figure 2.3

In considering this explanation some of the teachers instinctively recognized that Valerie's argument was based on the meanings of the concepts involved and the connections between these meanings, whereas the other arguments involving the calculator or the division procedure were based on following rules that for many in the group, were rather arbitrary. Discussions around this and similar problems were helping to illustrate the difference between conceptual understanding and a purely procedural knowledge of math. Coming to see that there is a difference is an important, and not altogether easy first step in the process of looking at math itself differently. Even then, there are necessarily many questions about how, or perhaps even if, this changing picture of the subject and its goals should affect one's teaching. As the teachers discussed their own thinking about problems, they related their experiences to that of their students.

Similar issues arose earlier in the week, while we doing some simple calculations in Base 5 such as finding the difference (32-14) and the product (23 x 4). These had brought out the fact that understanding place value involved not only dealing with groups of 5's or 25's et cetera, but also thinking about how these groups interacted in the different operations. Figuring out how to write Base 10 numbers in a new system was a challenge,

but one that seemed manageable. However, calculating in the unfamiliar setting of Base 5, trying to find a rule to determine which Base 5 numbers were even, and looking at how to write 1/3 as the equivalent of a 'decimal', helped us take a fresh look at the way our number system works and examine the wealth of ideas that lie below the surface of a computation. This effort was initially frustrating, but as the teachers used colored chips or other materials to represent the different place values, and drew diagrams to explain their work, they saw that their own uncertainty and problems in calculations reflected some of their students difficulties. This recognition helped us to talk about the thinking that was behind some of the errors made. Moreover, not having a ready procedure to apply in finding answers to the above questions forced us to consider both the concepts involved as well as different approaches to finding answers. And the discussion around this work led us to share some of the following questions about elementary classroom methods and goals.

- Using concrete objects or manipulatives had helped some teachers overcome both their anxiety and uncertainty about how to begin to look at (23 x 4) or (32-14) in Base 5. Perhaps using manipulatives in Base 10 would benefit their students. But how would you use them effectively? Was this simply a classroom management issue, or did we need to consider what one was getting out of the model mathematically?
- One way to represent a ten was with a single red chip that could be traded in for ten white chips, or ten ones. Another model used ten interlocking cubes for a ten, and a single cube for a one. Mathematically I preferred the former model because it offered a clearer picture of the place value concept with the single chip representing different amounts depending on the color used, but a number of teachers preferred the interlocking cube model because the students would really see the difference between a ten and a one immediately. Were different models more appropriate at different age levels? In addition to what a particular model illustrated, how did the use of manipulatives or diagrams and pictures in general relate to the ability to work symbolically with an algorithm? What connections did one need to make and how would one go about doing that?
- Working on problems together we had certainly seen that other procedures or algorithms were possible for carrying out calculations. These different approaches gave an intriguing picture of areas to explore in a topic that had previously seemed pretty rigid and straightforward. Yet, there was uncertainty in this broader arena too. Aren't some algorithms better than others, perhaps more efficient? Don't we want efficiency, or at least at some point? How do we know if a student's algorithm is correct?

Throughout the week these questions kept popping up as we moved back and forth from solving problems ourselves to talking about teaching and student learning. The process of using one's own work on some challenging math questions as a base for considering both the math and questions of learning and teaching was powerful. Diane, one of the teachers in the group described her experience with the following words.

> "My understanding of the subject [number systems] has changed - there is more to understand than I realized! I have been caught up in the computing of a

problem rather than reasoning it through - exactly the way I was taught. There is a much greater thinking process involved instead of just memorizing steps to come up with an answer. I now realize that there are many ways to get answers - and they are all right! There is more of a discovery process to mathematics - and this needs to be a part of our curriculum. ... I fear we have taught them [our students] to be good computing robots - and the thinking process which is most important has many times been over-looked or passed over quickly. Hopefully I can find a good balance of this in my own teaching."

Diane's comments focused on the role of mathematical thinking and developing understanding of math concepts. Some teachers in the group used their experiences to consider some of the social aspects of the learning process in general. Dealing with new concepts can be frustrating. One teacher wrote,

"I think it helped me to realize that base 10 might be just as difficult for one of my students as Base 5 or 12 would be for me."

Another said,

"At first I didn't think I could work in Base 5. I got a great feeling of what it's like to move too fast - great frustration." But then continuing later on she wrote,"I enjoyed the class even though it was very hard for me and I felt frustrated."

Others saw natural learning benefits from working in small groups, trying to explain their thinking, and trying to make sense out of someone else's method of solution. Although the sessions on Fractions and Number Systems were more closely tied to the familiar topics in the math curriculum, the work on estimating π, the area of a circle of radius one, or on the Pythagorean Theorem had the benefit of presenting a broader arena for the process of doing math as teachers examined their own reasoning while they worked with right triangles and discussed the concepts of area and margin of error. One teacher wrote,

"I've never gone beyond having kids find their first estimate. Often this ends up not being very close to the exact answer. I see how much can be learned mathematically by refining and working with margins of error. I see that so much more is learned and understood without drilling calculations. The calculations we did grew naturally out of what we were doing."

Towards the end of the math sessions (the first two weeks of the workshop), we could see that ideas about math as a subject were beginning to change. There was a growing awareness that in math too, one could discuss concepts, do experiments, that one could learn more by examining several approaches to problems and considering why rules worked. For many participants this new picture offered a sense of freedom and excitement, along with a feeling of accomplishment in the challenges they had met. But a number found our math sessions either irrelevant to their teaching, or too complicated to contribute to their own learning. Looking back over those last two weeks, we thought about what we could

do differently to make the experience more accessible to these participants as well. At that point it was hard to say. The level of the math sessions felt right to us (we were to see more evidence of this during the school year) but we wanted each teacher to derive some benefit from their summer work. Was this possible? We'd organized what we'd hoped would be several different ways to support the teachers' efforts and address their concerns as they tackled demanding problems. As we moved through the daily math sessions, we'd adjusted our pace and focus to interact with the classroom issues the teachers presented. We'd learned a lot ourselves in doing this! Still, there was more to understand about effective ways to bring together the teachers' pedagogical questions with the math work we felt important. Would the focus on classroom practice during the second half of the summer workshop perhaps offer these teachers a different way to examine some interesting mathematics? How would the ideas we had been considering about math and about teaching math play out in the classroom? We would all be considering these question as we watched Deborah Ball from Michigan State University give demonstration lessons and lead debriefing sessions during the third week, and as the teachers tried out lessons on topics from the math sessions during the fourth and last week of the workshop.

Talking about Teaching and Analyzing Lessons

The night before her demonstration lessons were scheduled to begin, Deborah Ball arrived for a meeting with the workshop staff. Deborah Schifter from the Educational Development Center also arrived that night and joined the discussion. Both Deborahs listened carefully and asked many questions as we described our work during the last two weeks. For each of the next three days Deborah Ball would give two fifty-minute classes, one for about 20 students from the 1st through 3rd grades followed by another for the same number of students from the 4th through 6th grades. Because of space limitations, and to maintain as normal a classroom atmosphere as possible, the teachers would take turns observing the lessons in the classrooms. The lessons would be videotaped so that those teachers not attending the lessons could watch them later on in the day. After her lessons each day, Deborah would lead two debriefing sessions, one for teachers of the 1st-3rd grades, and one for teachers of the 4th-6th grades. Deborah Schifter, as the internal evaluator on the project, would be our "extra eyes and ears" during the next few weeks as she helped us assess our efforts from the vantage point of her experience in similar work.

It is an intricate task to choose the kind of lesson or problem situation to present so that the students' work in mathematics will offer fruitful grounds for a discussion about the teaching of mathematics. The very nature of the interaction makes the outcome difficult to predict. But we were confident that in Deborah Ball we had someone highly skilled in handling this uncertainty. We tried to fill her in on the backgrounds of the workshop participants, the nature of their interests and concerns, and on the students she would have in her classes. What does one hope to gain from such an exercise? At its best, experiences like this can lead to a natural flow of ideas about the mathematical concepts, the teaching questions, and the students' thinking. And they can both strengthen and enlarge one's

understanding of the connections between these issues. For this to take place however, one must step back and consider the role of teacher and student in the classroom, long-range goals of learning, and the effects of different approaches towards reaching these goals.

For most elementary school faculty there is neither the time nor the encouragement to think about their professional activities in this way. Faced with the immediate concerns of the school day and frequently isolated from their colleagues, there is little opportunity or incentive to reflect on their teaching beyond the next few lessons. There is little chance for teachers to observe one another in non-evaluative situations, where the purpose is a collegial exchange on the practice of one's profession (Heaton, R., Lampert, M., 1993).

Having little experience with this kind of reflection, how would the workshop participants use the demonstration lessons and debriefing sessions? We knew the teachers were looking forward to watching these lessons, and we were hopeful that the next few weeks would offer us all another context in which to examine questions about mathematics and goals in teaching mathematics.

The children, our students for the demonstration lessons, began lining up early on Monday morning to sign in, receive a name tag, and find their classroom. The first lessons began with Deborah Ball spending some time learning the children's names and explaining how they would work together in the class. She introduced in each group a problem situation in which the children, working with some manipulatives such as square, colored tiles, and colored cardboard circles called 'chips', would be encouraged to think about whole numbers and their relationships. For the younger grades, Ball had a board divided into four squares which were to represent the numbers 1, 2, 4, 8. She called this board a 'mini-computer'. One could represent the number 7 by placing one chip on each the squares representing 1, 2, 4 since 1+2+4 = 7. Of course that wasn't the only way if more than one chip could be used on a square, and Deborah would ask the children to investigate the different ways they could represent the numbers 1 through 10. After Ball introduced these problems to the group as a whole, she asked the students to begin working individually or with a partner. Circulated among the tables, she talked with the students, answered questions, and encouraged the students to record their work with drawings or explanations in the small notebooks, or journals, she had given each student. After some time spent on their investigations, Ball would initiate a discussion with the whole class about the problem by asking students to show some of their work on the blackboard and to compare their results. In the upper grades the students used color tiles to examine factors of the whole numbers through 20. For example, with 12 color tiles, how many different rectangles can be made if each rectangle uses all the tiles? From their results they would look at special properties of these numbers. At the end of the days' lessons, Ball met separately with the teachers in each group for a debriefing session. These group discussions were an opportunity for the teachers and Ball to examine the events in the lessons and consider questions that these events raised about teaching mathematics.

During the next few days of watching the demonstration lessons and debriefing with Deborah Ball, the participants noted various aspects of her teaching such as the use of

journals and the combination of individual and whole group work, the emphasis on discussion, the fact that she spent a whole class on a single question. They were intrigued that she accepted all responses to her questions with little comment, instead asking the student to explain the reason behind the answer and expecting the class as a whole to consider the appropriateness of the response. This requires more effort from the students (not to mention the teacher!) and the teachers were both pleased and surprised at the performance of students with whom they were familiar. Many appreciated the effect this approach had on the children's self-esteem, and some also had a glimpse of the potential this offered for learning. There were questions too. The pace of the class was slower. Was this why some of the students were not engaged? Would such an in-depth look at a given topic prevent one from covering the required curriculum? Wasn't there too much emphasis on developing understanding? Perhaps more would be gained by emphasizing skills first, then later on focusing on understanding.

Deborah Ball had begun the debriefing sessions by explaining that as a part of her research at Michigan State University, she taught the math class every day in a nearby third grade classroom. Just as at Washington Elementary, many of her third grade students spoke English as a second language because they were children of foreign students at the university. Also interrelated was the circumstance that the classroom composition changed frequently during the school year. After that opening she explained that she was very used to discussing her lessons and she invited the teachers to offer any comments or questions they might have on the lessons. Not quite certain where to begin, the participants focused initially on general procedural questions or comments about how to use journals effectively or encourage and assess group work. Gradually they become more at ease over the three day period as they began to see that Deborah not only welcomed their comments but then used these comments to share with the teachers some of the questions she was asking herself, questions which occurred to her in the middle of the lessons. As Deborah invited the teachers to help her examine particular responses given in the day's classes, the discussion shifted to include consideration of the students' thinking and relationships to varied aspects of the mathematical concepts involved. Deborah made it very clear to the teachers that she saw her teaching as an on-going process of inquiry, one that did not necessarily have a fixed model as a goal. This outlook presented a very different perspective for many on the profession of teaching, but it was perceived by some at least, as one that would allow us all to grow in the process of teaching mathematics.

Who Is Responsible for What?

Towards the end of the summer workshop I read over the teachers' comments on our joint activities and the notes I had made on our work together during that time. The image of math as a subject dominated by memorized, mostly inexplicable, rules had been challenged. Many of the participants were excited by a broader picture of the subject. I was impressed with their enthusiasm to put some of their new ideas about teaching math into practice. At the same time however, I was concerned about several issues. The teachers

new understandings about the subject of math were built on just a few learning experiences, which although powerful, were not necessarily extensive in scope. How would these ideas be implemented? For example, what effect, if any, would the work on connections between fractions and division have in the classroom? The fact that estimation naturally involves different approaches and/or different answers appealed to a number of participants because of the greater opportunity to encourage all reasonable student efforts. Yet, would this lead them to take the next step of comparing approaches and considering ways to make better estimates? Teachers had begun to see that asking questions about a student's answer, rather than simply deciding if the answer was right or wrong, resulted in more active students thinking. But, as one teacher wrote,

"Questioning that provokes thought is hard to do."

One can ask a student to explain their work, but taking their responses and using them well requires both mathematical knowledge and perspective, especially on how the subject develops across the grades. These understandings need time to grow and a four week summer workshop is only a start!

On top of all these concerns, we wondered what kind of experiences the teachers would encounter as they made changes in their teaching practice. Worrying that they might try too much too quickly, we reminded them that change begins slowly and develops in small steps. From our own experiences in university classes, we were aware of some of the difficulties one can encounter in trying out new teaching strategies, even in the situation where the subject matter was our chosen specialty. The teachers we were working with were assuming a much bigger challenge. They were attempting to revise their own understanding of math and, at the same time, make changes in the way they taught math, all of this with a daily schedule that included little time to thoughtfully examine the results of their efforts. What kind of support could we offer them, what sort of contact would be useful, and how could we continue the promising conversation about mathematics that had begun during the month of the summer workshop?

Classes were only a week away as the workshop ended. Carol, Herb and I met to discuss the school-year support that the project had promised to provide. Herb and I had discussed this in the early stages of project development, and I had agreed to be a "math consultant" for the school faculty during the school year. The intention was that I would spend two weeks in each teacher's classroom as they worked on implementing a theme they had studied in the summer workshop. During this period I would help the teacher plan lessons, offer feedback on student work, and help analyze what worked or didn't work in the lesson. Lorel Preston, a graduate student in mathematics education, would assist me in this work. Together we would plan visits and be available by telephone for the teachers, offering them a "math 911" number. In our initial planning for the school year work, we had placed a lot of emphasis on teachers taking the initiative. They were to contact us for support as they began teaching the themes they had studied in the summer. Now, as we talked about the school year ahead, we all realized that this was unrealistic for several reasons.

During the past month it had been hard for us to break out of the usual mode of university faculty leading discussions and organizing activities, and hard for teachers to break out of the mode of seeking our direction. Towards the end of the workshop, we asked what kind of support the participants would find helpful during the school year. But we received only a few suggestions. Working together in a summer math session is one thing, but working together in the teachers' classrooms was quite another. Not having been involved in such work previously, the teachers were uncertain about how we might be useful in the classroom and what kind of interaction might develop. So, working with Carol, we set up a framework that would help define the school year support and encourage the teachers' continued involvement in the project despite the daily pressures of the classroom. We made an initial plan that we would try out until November 1st. At that time the faculty would have their first professional day (release time from teaching) and we could all evaluate how the plan was working. Ultimately we wanted to share equally with the school faculty the responsibility for setting the direction of our work together.

In making our plan there were two activities that seemed very important: 1) to work with individual teachers in their classroom, and 2) to continue the group discussions that had begun during the workshop on math content and pedagogy. We set up one day each week to hold morning meetings with teachers by grade level, first through third grades one week, fourth through sixth grades the next week. The school would hire substitutes weekly to cover the teachers' classes during their meeting time. I also arranged to be at the school two other mornings each week to visit classes and consult on math questions. In addition, Lore would attend the grade level meetings and spend one morning a week at the school observing in classrooms. With this plan, we were able to organize a forty-five minute meeting every two weeks at each of the first through sixth grades. We would also be able to visit about six to eight classes each week.

In order to give the school faculty time to relax after the long summer workshop and to get their classes underway, the school year program would begin about a month into the school year. Also this would give us an opportunity to think back over the summer workshop and consider how to begin our meetings.

Although this plan seemed to be a reasonable way to organize our work, there were a few important questions to consider. First, we had to ask ourselves, would the teachers want us to visit their classes, and could we develop a basis for analyzing what was happening in math lessons that did not put us in the role of evaluator? How could we make our visits useful? During the summer program teachers had given some practice lessons and debriefed with us and their colleagues afterwards. It was challenging both for us and for the participants to give meaningful feedback that was sensitive to a teacher's efforts and goals, and that would lead to continued growth rather than to discouragement. In describing this activity on the exit survey, one teacher reported that the small number of comments s/he received led to concerns that perhaps the lesson had been too awful to comment on! It seemed to me that we were asking a great deal of the teachers and that we would need to develop a closer working relationship first and to think carefully about our role as observers. Coincidentally, my colleague Jennifer, who had participated in the

summer workshop, asked if she could sit in on my math course for preserves teachers. This seemed like a good opportunity for me to experience having a regular observer in my classes and to consider effective ways for us to debrief with each other.

Secondly, I felt we needed to have of a focus for the biweekly group meetings, one that would allow the faculty to share their lessons with each other and analyze what worked or didn't work and why, but also one that would help us all to consider school-wide math goals. One of Deborah Shifters suggestions from her evaluation on the summer workshop seemed like a good place to start. Some of the teachers were beginning to sense that there was a 'big picture' of mathematics that they were missing and that it was this picture that Deborah Ball used in making up her questions and problems for her classes. To help develop this picture, Deborah Schifter suggested that we ask the teachers to set priorities for their math curriculum for the year under the categories of 1) topics 2) habits of thinking 3) big ideas. (For example at the second grade level a *topic* might be addition and subtraction of two-digit numbers, checking on the reasonableness of answers by estimating a ballpark figure would be a *habit of thinking*, and the inverse relationship between the two operations could be a *big idea*.) Deborah Schifter often used this exercise with the participants in her summer institute for teachers, Summer Math, at My. Holyoke. These teachers had found the exercise useful, reporting that it "helps them to start thinking in terms of taking control over the curriculum."

Examining this broad question together would give us a natural context in which we could consider math content and pedagogical issues. Working with the whole school faculty was going to be a challenge, but sharing ideas within and across the grade levels could help us develop some perspective on the math curriculum as a whole. We were uncertain about how our plans would turn out, but this seemed to be a reasonable way to begin the next stage of the project.

CHAPTER 3

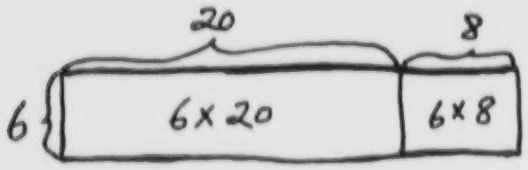
BUILDING TRUST AND WORKING TOGETHER

What support during the school year is most useful to teachers as they try out ideas from a summer workshop? Can mathematicians be useful in an elementary school classroom and what can they learn about teaching that will benefit mathematics courses for prospective teachers? What mathematical connections are important between grade levels and how can these be developed? This chapter examines these and other questions by describing how school and university faculty developed ways to work together in classrooms and in grade level meetings. (September 1992 - January 1993)

It was towards the end of October when our third meeting with Angela, Scott, and Jackie, a group of fourth and fifth grade teachers, took place. Angela began the discussion by describing one of her student's work on the following problem. If 70 children share 350 candy bars, how many bars of candy will each child get? Although the students had done similar problems using division with one-digit divisors, and knew that dividing 350 by 70 would give them the answer, Angela said that the class had not yet studied any rules for 'long division', or division with two-digit divisors, so they were uncertain how to get the answer. She was delighted when one student suggested that since $350 \div 70$ meant "how many 70's in 350", they could subtract 70 from 350, then subtract 70 from the remainder, and continue this way till they could no longer subtract 70. The number of 70's they had subtracted would be the answer they wanted. Angela described the class discussion around different ways to carry out the subtraction, pointing out that it was the student's understanding of *what the operation meant* that allowed the student to do the problem without knowledge of the usual long division procedure.

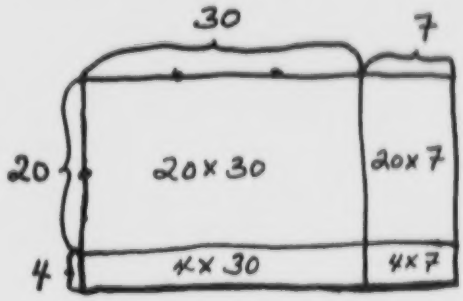
Angela's example and her emphasis on the student's understanding of the concept of division surprised me. Back in September we had begun to consider what the important math concepts might be for this grade level, but we were still struggling to understand each other's goals. Angela had suggested that knowing the multiplication facts was one of the big concepts for this grade level. I had said that I considered this knowledge to be very important too, but that I saw it as only one aspect of what the students needed to learn about the operation of multiplication. Angela had disagreed with my comments. Both she and Scott felt the basic knowledge of these facts was a major goal for the fourth grade, and all three teachers were doubtful about my ability to understand the difficulties of developing students' computational skills. I was to learn more about these difficulties as time went on, but I felt it important for us to consider a broader picture of multiplication and computation in general. This picture would include memorizing arithmetic facts, but I didn't want to see the focus on these facts hinder us from examining the meanings behind the computations: why certain procedures worked, how operations were related, and what this implied for their students' learning. I was hoping that together we could find effective ways to promote both technical skill and understanding in computation. But as a group we had not yet reached a point where our discussions seemed productive.

Now, Angela's introduction presented me with a good opportunity to ask what pictures of multiplication, or ideas about this operation would be useful for the students considering products like 6 x 28, and 24 x 37. Thinking along the lines of Angela's example, it seemed reasonable to the teachers that if a student knew 6 x 28 meant a sum of six 28's, then the student could find the product using addition without knowing a multiplication algorithm. This idea of repeated addition wouldn't be very efficient though, especially for something like 24 x 37. Who would want to add twenty-four 37's? At this point I suggested looking at these products using the area model shown below in Figures 3.1 and 3.2



28
×6
48 -> 6x8
120 -> 6x20
168

Figure 3.1



37
×24
148 -> (4x7 + 4x30)
740 -> (20x7 + 20x30)
888

Figure 3.2

The teachers were intrigued by this geometric picture of multiplication and its connection to the steps in the algorithm as we found the areas of the indicated regions. In the simple case of 6 x 28 (Figure 3.1), one could see clearly that to find 6 x 28, or the sum of six 28's, you could first find six 20's and then add this to six 8's for a total of 168. Breaking 28 into (20+8) made the process easy to compute mentally. Using this idea of place value and the Distributive Property ($a(b+c) = ab + ac$) in the second example, we could see how the familiar partial products of the algorithm emerged. But how would this work with their students? Jackie felt that keeping their students' attention as they worked through the stages of the above drawing might be difficult. What would the students gain from looking at multiplication this way? Should the students compute products this way first, or come back and look at this diagram after they had learned the algorithm? This approach to multiplication would take some time to develop. Would it be worth it? As the teachers considered these questions and looked at the diagram, they noted that by rotating the rectangle, the diagram clearly showed why 24 x 37 = 37 x 24. They realized this is not at all obvious when one thinks of 24 x 37 as a sum of twenty-four 37's, and it is an especially useful property in memorizing multiplication facts. We continued to look at the diagram for the rest of the meeting and discussed ways this area model might be modified to look at multiplication of fractions and decimals. Having a model that one

could use as ideas about multiplication grew across the grades was useful. The steps we used in Figure 3.2 above to determine 24 x 37 by writing this product as (20+4) x (30+7) and then finding the four partial products, are exactly the same used in showing that the more abstract product $(a+b)(x+y) = ax+ay+bx+by$, shown in Figure 3.3. Scott could see the benefit here for his students, many of whom would take pre-algebra in the sixth grade.

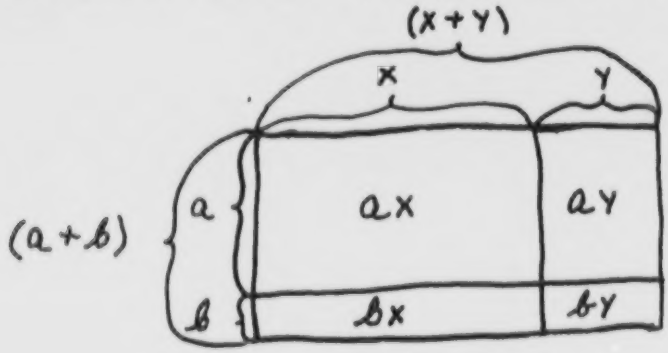


Figure 3.3

As the meeting ended, I felt for the first time that we were making progress in our ability to look at a math concept together and share our ideas about the content and pedagogical issues involved in that concept. Although our conversation about teaching mathematics during the summer workshop had wound up on a very upbeat note, we were all wondering how to continue this discussion within the classroom during the school year. The school faculty, and Lorel and I, each brought very different experiences of mathematics and of teaching to our discussions. It would take us awhile to understand how these differences affected our goals, our ways of interacting, even our style of conversation. We would struggle with ways to make our interaction fruitful. Many teachers would be uncertain about the purpose of our discussion of math concepts and would ask what connection this had to their teaching practice. Was it really so important to consider one's own, or one's students', ways of thinking about division? How would this help one teach division in a better way? Why, if I was the expert in math, didn't I simply say how best to teach a given math topic? Yet for me, there *wasn't* a 'best way' to teach a topic. There were, to be sure, aspects of a topic (such as the use of the Distributive Property in multiplication mentioned in the above example) that I considered very important for mathematical growth. But topics can be examined in a variety of effective ways. Moreover, there were to be sure, changes in teaching that I hoped to encourage the school faculty to consider, such as placing more emphasis on the reasoning aspect of mathematics and the relationships between concepts studied. But I didn't have a particular belief as to how this could or should be done. What I hoped to do instead was to establish a climate in which we could examine these questions together, a climate that would support each individual teacher's potential for growth as a math instructor. Moreover I wanted the paths towards change to be set by the teachers, not by us. For this to happen, I would have to learn about the faculty's goals for students, goals that go beyond the mathematics lesson. I would have to think about the teachers' reasons for pedagogical decisions made in their math lessons. At the same time, I felt compelled to offer a long-range perspective on the subject that the

teachers would find challenging and useful. At the beginning, learning from each other's experience would involve some anxiety and uncertainty for all of us. Gradually however, we would came to understand that this tension is a necessary part of jointly determining the direction for our work and sharing the responsibility for reaching our goals.

Learning about the School Culture

Looking back after the first month or so of visiting classrooms and holding meetings (mid-September till near the end of October), I was quite uncertain how our work at the school would develop. In the meetings, it was difficult to focus our discussions on what were, to me at least, the important issues about the math curriculum. At our first meeting with each group of teachers, we had suggested developing an outline of the grade level goals, or as we came to call it, a 'mathmap' for the grade. I had thought this exercise would give us a natural context for sharing ideas about math content and pedagogy. But despite the fact that the teachers also thought this was a good way to begin, we were making little headway.

When I had asked about visiting classes, a few teachers invited me and my graduate assistant, Lorel, to drop in anytime. Most others asked us to set up a time for a visit. During these visits we noted several teachers trying out some ideas they had worked on in the summer workshop. But we had trouble at first finding ways to reflect together on these experiences. Within the teachers' daily schedule there were only a few moments between classes to look back at the lesson and reflect. And too, we were all a bit uncertain about how to productively discuss these lessons. Nevertheless, through the grade level meetings and classroom visits, we began to learn how the school math program was organized, what goals the different teachers had for their math classes, and what were important factors that we would need to consider in our work at the school.

It is probably appropriate to consider some of these factors at this point. First, the students at Washington were grouped by ability level for their morning math and reading classes. It had taken Lorel and me awhile to understand how this worked. Generally speaking, from second grade on, a student's math group was determined by computational skill demonstrated on a written test given during the first few weeks of the school year. Students were grouped somewhat across grades, resulting in roughly two or three ability groups in math at each grade level. In addition to these groups there were also two ESL classes, one for the first through third grades and one for the fourth through sixth grades. This large number of groups was made possible through the use of the Title One and ESL teachers, as well as teacher aides as instructors. The morning math classes were intended to focus on number concepts and on developing computational skills. It was left to the homeroom teacher in the afternoon to work on other math topics such as time, measurement, money, geometry.

The teachers had carefully developed their plan a number of years before. They knew that the practice of grouping by ability level was currently criticized, but they believed their plan had many benefits. Although we were dubious about the effectiveness of this

plan, this was an area in which the teachers had the greater experience. It was up to us to learn from them about the practical aspects of dealing with a highly diverse student population. When asked to describe the advantages of this extensive grouping, they made the following points. The fact that all of the students changed their groups for math and reading lessened some of the social stigma of the more typical pull-out programs for Title One students. Also, these students did not miss their regular classroom work and were with their classmates for the majority of the school day. By reducing the student-teacher ratios through the addition of the Title One and ESL teachers, the majority of the faculty felt they were better able to meet the needs of this student population with its widely varied academic background and a high turnover rate. At this point I had too little actual information to know how the plan affected the students, but I wondered if the extensive grouping didn't contributed to a fragmented picture of the math curriculum and a lack of common goals. Also, the division of the math curriculum into the morning focus on computation and the afternoon work on "the other math topics" only emphasized, for both teachers and students, the idea that the main goal in math was to do arithmetic calculations. Moreover, not all classes did do math in the afternoon. Whether this was due to lack of coordination in planning curriculum or to a sense that 'afternoon math time' was an 'extra' that could be dropped when pressed for time was unclear. If computational skill is the main purpose of a class, then rote memorization of algorithms may oe the most efficient way to achieve this goal. From that point of view, our questions about developing conceptual understanding, connections between concepts, or consideration of deeper and more extended problems seemed irrelevant.

There was another factor that hindered our discussions too. From what we could see only a few teachers were examining their math curriculum in preparation for the grade level math meetings. The practical concerns of what to teach next and classroom management issues were uppermost in many teacher's minds. It was all too easy to spend our brief time together, 45 minutes every two weeks, on these immediate issues. I was gradually coming to see how important these issues were too, yet we also needed to examine broader questions about math goals if the mathematics program at the school were to improve. I had thought such questions would arise naturally as we developed mathmaps together. But at this point I could see that taking a long-range, in-depth look at these goals was primarily of interest to Lorel and me, not to the teachers. Although developing mathmaps seemed important, I did not want either these maps, or the direction of our discussions to come from us for several reasons. To be effective, our work together had to make sense to the school faculty. It had to be developed around ideas that they believed important, ideas in which they had invested their thinking and efforts. Otherwise it would ultimately have little impact in the classroom. Looking back now, it is easier to appreciate that we were asking a lot of the teachers. They normally followed district guidelines or used their textbooks. We were asking them to take up the task of examining their math goals in the light of a new and uncertain picture of mathematics that they had developed during the summer workshop. This is a lot under any circumstances, but especially so when teachers are rarely given this kind of authority or responsibility.

Discussing these concerns together with Herb, Carol, and Lorel, I came to realize that

developing mathmaps was probably too large an issue to handle at this point. If we wanted the active involvement of the teachers in setting some direction for our discussions, then we needed to use their immediate classroom concerns as a focus. So, at our next meetings I suggested that we spend our time developing a lesson together on a single topic, one that the teachers in the group were presently working on in their class. To prepare for this round of meetings, Lorel and I decided to send a memo to each group prior to the meeting. The memo would state briefly what had been done at the last meeting, and then suggest a few questions to consider next. In this way we hoped to maintain some continuity in our discussion over the two-week interval between meetings. Gradually we found that this smaller focus around current classroom issues helped the teachers connect their meeting discussions and classroom work. (This is described in detailed examples in the next section.) It allowed us to bring up what we saw as important math content issues within the context of the teachers' daily work. Then together we could begin to relate these issues to long-range planning for their classes.

Just as our roles in the grade level meetings were evolving, we were also trying out different ways to work with the teachers in their classrooms. As we got to know the teachers better, more were comfortable with us just dropping by their class without prior notice. The students too became used to us appearing and taking a seat next to them in the classroom. It had been very important to me to try to find a way to ensure that neither Lorel nor I were seen as evaluators during these visits. Some teachers were beginning to ask us to clarify a math point in a class, or to invite us to ask a question of the students, or to work with a group of students. I knew it would take time to place the focus on 'what went on in the class, what worked well, or didn't work well, and why', and not have it be a question of 'how well did I do'. I had remembered a comment of John Dossey, a former president of the NCTM, when he was asked how mathematicians might be of help to school teachers in their mathematics classes. He had emphasized that the best way to begin would be to simply sit in the classes, watch and see what went on, and try to understand the goals of the teacher and the way the class worked.

This made a lot of sense to me from having my colleague, Jennifer, sit in on my own class at the university. We had similar backgrounds and experience, but it had taken us several sessions to comfortably share our thinking about what went on in the class. What had helped was to begin by considering the students' response to one of my questions. Comparing our understandings and impressions of the students' work led us to consider alternative questions and approaches to the course material. Explaining to Jennifer where I was trying to head in the class and why helped me refine my thinking about my goals for the course. With her different perspective as an observer, she was able to help me examine the effect of the plans I made. It was this kind of experience that I hoped we could provide for the teachers. So, talking with teachers after a lesson I usually began by asking about a student's comment. In the few minutes we had between classes we would consider what this implied about the student's thinking and then, if we wanted more time, I would call the teacher at night to continue the discussion. In our conversations with the teachers Lorel and I wanted to be a sounding board for the teachers thinking about their classes,

and to support any efforts we saw being made to help students think more conceptually about mathematics.

Gradually over the next few months we were to see our conversation about teaching math begin to blossom in the meetings and in the classroom visits. As our work in the project evolved we saw more clearly the value of allowing the direction of our conversation to develop slowly with the help of all involved. We came to understand that the early uncertainty about how to proceed was a necessary step in developing a partnership.

Building a Dialogue around Classroom Events

Working closely with elementary school teachers, I became aware of how the typical daily schedule works against a teacher's efforts to reflect on their teaching and to collaborate with colleagues on academic issues. In comparing their situation with my own, I was impressed with the fact that it was precisely these activities of reflection and collaboration which I considered to be a major part of my role as a teacher at university. With the exception of a brief lunch (usually a half hour) and perhaps one to two hours of free time during the week, K-6 teachers are constantly responsible for organizing and directing the activities of the twenty-five or so young children in their classroom. In the best of circumstances, being actively present to the needs of others for long periods is a draining experience. In today's inner-city schools, this drain has intensified as the social and behavioral problems have increased. With only a few hours for planning lessons (generally about three) built into the week's schedule, the structure of the elementary school program discourages teachers from thinking critically about their classroom practice, the very activity that plays a major part in teaching well and in continued growth as a professional. As a principal, Carol was very aware of this problem. She knew that in the face of these immediate demands, it is difficult for teachers to see their own learning as important. With this in mind Carol encouraged the teachers by making it clear how much she valued their work in the project, by organizing the day's schedule to establish meeting times, and by allocating school money to pay for substitutes. Although the Washington faculty were concerned at first about the impact of the grade level meetings on their students and on the classroom routine, they came to value their time together. The meeting time itself seemed very little, but the fact the these discussions continued to develop around events in the classroom began to offer several teachers a new perspective on their classroom work.

Several months into the school year Lorel and I met with Suzanne, John, and Sally. This group of teachers were working on addition with their mostly second grade students and this topic had been the focus of our discussions in the previous meetings. John had a group of about twelve or so resource students, including some with either learning or mental disabilities. Sally had the students who were on grade level in math, and Suzanne had those who were above grade level. Both Sally and John had attended the whole summer workshop, but Suzanne had missed the first two weeks of math content work. When we met for the first time in the fall we learned that Suzanne and Sally had begun the year by looking at addition facts to 18. They were collaborating in using the Math Their Way curriculum, a program that Suzanne had learned about in a workshop and which she had

followed during the previous year. John had also planned to begin with addition facts, but he had quickly realized that his students had very little understanding of what numbers up to 20 represented, or how to count on in simple sums.

The teachers had spent several meetings talking with me and Lorel about understanding number symbols, how this related to the operation of addition, and the different ways that children might begin to develop number sense. Understanding place value is clearly important, but exactly what does that mean? For example many of the students could say that in 32, the 3 stood for 3 tens and the 2 for 2 ones, yet the teachers knew that ordering the numbers 32, 29, and 41 would not be a simple task and they were very familiar with the common second grade mistakes in addition and subtraction shown in Figure 3.4 below. What did this say about the students' understanding of place value and what did it mean for the teachers as they considered how to approach this topic?

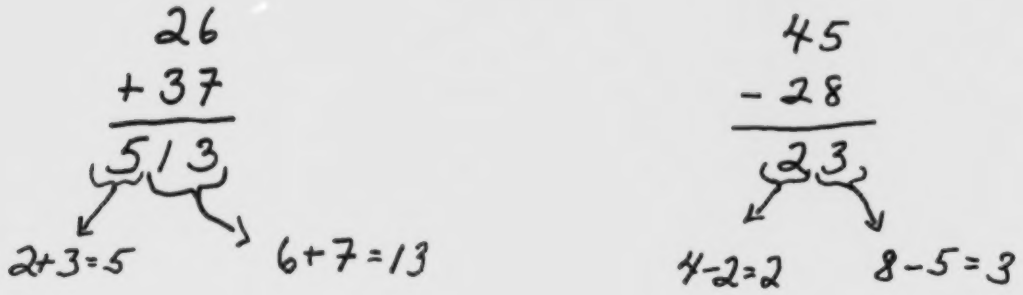


Figure 3.4

Suzanne and Sally thought that by using a single bean to represent a one and a cup of ten beans to represent a ten as described in the Math Their Way program, they would help the students see the number 32 and the relative amounts represented by the 3 and the 2 'concretely'. Then they could use these concrete objects, or manipulatives, to demonstrate how the steps in the standard addition and subtraction algorithms worked. In preparation for this step, Suzanne's and Sally's classes were memorizing their addition facts to 18. They were using some aides such as doubles like $8+8$, leading into doubles plus or minus one such as $8+9$, or $8+7$. They were both pleased to see the students growing recall of the addition facts. John had decided that he would leave addition facts till later and explore place value first. He had invented a game for that purpose. He represented units by an animal called a chickadee and said that a tribe was a group of ten chickadees. John would make up adventure stories which took place in a forest and involved adding and subtracting small amounts of chickadees. Throughout the stories the students kept a running total of the number of chickadees present by mentally doing sums and differences. They reported the number of tribes and left over chickadees to John who would record

the number symbols on the board. Although pleased with the students growing ability to count on or back by small amounts and their grasp of the concept of a tribe, John felt they needed more experience with grouping before he moved on. So he developed a game around Base 3 numerals that involved using poker chips to represent amounts. He began having the students themselves do the recording with written symbols. John felt that working with a smaller group size, 3 versus 10, would make it easier for his class of special needs students to study the process of grouping, especially for those students with memory problems. Also, the work would involve fewer chips and thus fewer management problems.

From visiting all three teachers in their classrooms between our meetings, Lorel and I were beginning to understand their different approaches to teaching and to the subject of math. Thinking over our meeting discussions and what we had observed, we saw a number of questions to consider as the three classes moved on to double-digit sums and differences. In learning the addition facts, which strategies are most helpful and why? To us it seemed important to learn you can find the sum of 8 and 5 by first making a ten, for example, by taking 2 from the 5 to make the 8 into a 10. A second step was then to see how much of the 5 is left. This idea of 'making a ten', along with the ability to see a number like 25 as 20+5, seemed to us to be very useful in finding the sum 25+18. This was also a natural way to calculate mentally. It was this way of using place value now in addition that we felt would help students build a flexible picture of whole numbers. This would be important for their later work, far beyond addition. The teachers were doubtful about this approach as natural or especially useful. Suzanne had seen that her students were able to learn the standard addition algorithm when she described it in terms of beans and cups. Shouldn't the students know the standard algorithm? What was the point of breaking up the numbers this way? Wouldn't it make addition more complicated? After we had been considering these questions for awhile, and just before we met to continue the discussion, I had sent a memo containing the following questions:

> What is the role of the following in learning to add: counting by ten (or by 3 in Base 3!); knowing sums to ten, and grouping in tens? Where do these concepts lead, and what thinking skills are they related to?

Sally and Suzanne brought in an example from their class that related to these questions. Sally had been reading Dr. Seuss's story about Gertrude McFuzz. In this story, as Gertrude eats berries from a certain bush, she grows more tail feathers. At one point the story says that Gertrude now has three dozen tail feathers. A boy in the class said that he knew how much three dozen was. When Sally asked how much, the child replied 36, and Sally asked him to explain how he knew that was right. The boy began by saying he could take 12 and make it into 10 and 2. Since he knew three 10's made 30, and three 2's made 6, he could put them both together and get 36. Sally and Suzanne were excited to see one of their students coming up with a process we had been discussing. They recognized the power this student's understanding of number offered him in answering a math question that had captured his attention in another context. This classroom example presented by the teachers resonated with our meeting discussion. It provided a much more meaningful

illustration of the role of place value in addition than one that we might have constructed on our own. Beans and cups were helpful in showing how to use the standard algorithms for addition and subtraction and to explain the reasoning behind these steps. But the teachers were beginning to sense that this approach by itself would not develop the flexible number sense the value of which their student had demonstrated in figuring out how many tail feathers Gertrude McFuzz had grown. Learning about algorithms, or efficient ways to obtain answers, is important. But if students begin their work with arithmetic operations by learning calculation rules first, are they less apt to think about numbers and how they work? Although it often seems improbable and indirect, playing around with numbers and their grouping builds a flexible number sense. How could one encourage this exploration and at the same time develop a certain level of competence in calculations? Exactly what was useful for the students in working with manipulatives and how do students relate the work with these objects to symbolic manipulation? Our sense of these issues gradually evolved during our meeting discussions. Bringing in other examples of student work, the teachers began to question previous assumptions about what students would find reasonable or understandable in doing mathematics.

In other groups too, looking at student work raised questions about what were appropriate math goals for the students. One group of teachers who had encouraged us to join their classes from the beginning were those teaching the students who were below grade level, including the resource and Title One students. These teachers held two classes, one for the first grade, the other for the 4-5th students. Louise, Peggy, and Jill (a part-time teacher) organized these classes. They were assisted by Cindy, a teacher's aide, and sometimes by Karen, the computer lab director. All of the teachers in this group had attended the summer workshop. They had found the math challenging but at the same time, they were excited by their own new learning. In the Number Systems and Geometry sessions, they had seen how useful it was for them to describe their thinking with manipulatives or with diagrams on paper. A number of these teachers had taken a Math Their Way workshop in the previous years. They had found that using manipulatives to illustrate concepts had benefitted their first grade class. Now they hoped similar efforts would help the students in the 4-5th class. Many of their students had either learning disabilities, language problems, and/or behavior problems. Keeping these students on task and attentive was a challenge. As judged by the computational tests at the school, the distance between them and their peers was steadily increasing. The teachers felt a lot of pressure to help these students close that gap. They were not sure that looking at the meaning behind math concepts would accomplish this, but they were willing to give it a try.

Louise and Peggy had decided at the beginning of the year that in their 4-5th class Louise would focus on conceptual understanding and math applications, while Peggy and Jill worked on computational skills. Dividing the class into two groups, they had one group spend about twenty minutes with Louise and Cindy, working with Base Ten blocks and drawing diagrams on paper to model numbers and operations. During this time, the other half of the students took timed tests on math facts and completed worksheets of calculations, supervised by Peggy and Jill. Then the two groups of students switched teachers. Employing a lot of individual attention in the small groups they had created,

the teachers hoped that both the students' understanding and technical skill would grow. Now, as they were looking at multiplication of whole numbers, they had run into several problems. Although quite a few of the students had worked on multiplication previously, it was clear from their worksheet efforts that very few could use the algorithm successfully.

We were talking about this problem in our meeting and I had asked the teachers how they developed the algorithm in the example 27 x 3. (This algorithm was also considered in the first section of this chapter.) They said, "multiply each digit by three". I did, and wrote 621, shown in Figure 3.5 below.

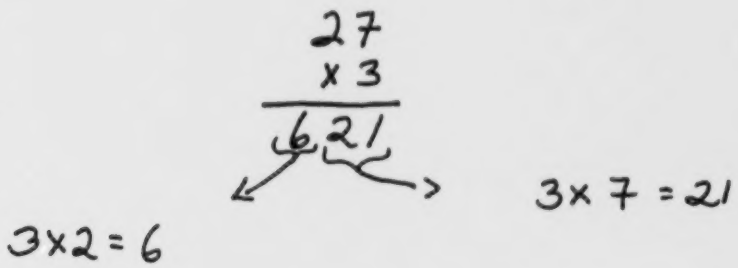


Figure 3.5

As I wrote this down, the teachers immediately responded that this was what many of their students did. It is easy to understand the answer of 621 if we ignore the appropriate assumptions about carrying. I shared some similar examples from my own classes with the teachers and we considered how much can be learned by looking for the reasoning behind student errors. Then I asked whether the students would see any contradiction between their answer of 621 using this method, and the idea that one could think of 27 x 3 as 27 + 27 + 27, which couldn't possibly be that large. In discussing this question the teachers thought that quite a few students didn't 'see' 27 x 3 as a sum of three 27's, but that they saw it more as a direction to follow a certain set of steps to get an answer. In doing worksheets of calculations, the students weren't used to asking if their answer was reasonable or not. We agreed that to be able to judge it is important to have a mental picture of what multiplication is. Moreover, as Louise pointed out, if one forgot the steps in multiplication one could still get the answer using the idea of repeated addition. She considered this a useful alternative for her students on standardized tests.

We had not yet settled the question of how to develop an algorithm for multiplication and we returned to this point. Jill suggested that we write down the previous problem in the diagram shown in Figure 3.6 below. As she set this up, she explained that the 6 would go in the tens position, because when you multiplied the 2 by the 3, it was really 2 tens and not 2 ones. Someone else insisted that we put in not just a 6, but instead that we needed to put in a zero as a placeholder as shown in Figure 3.7. Others disagreed and

for a few moments the discussion centered on which steps were allowed and what was the best way to describe them for the students.

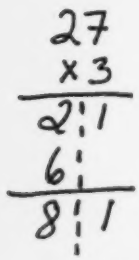


Figure 3.6

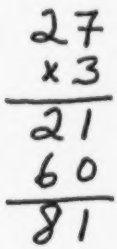


Figure 3.7

Then, Cindy asked Lorel and me what we would do. We suggested that perhaps by looking at 27 as $20 + 7$ we could see more clearly where the 6, 0, and possibly the 60 came from. Then, as shown below in Figure 3.8, we could see that 27 x 3, or three 27's would be the sum of 60 and 21.

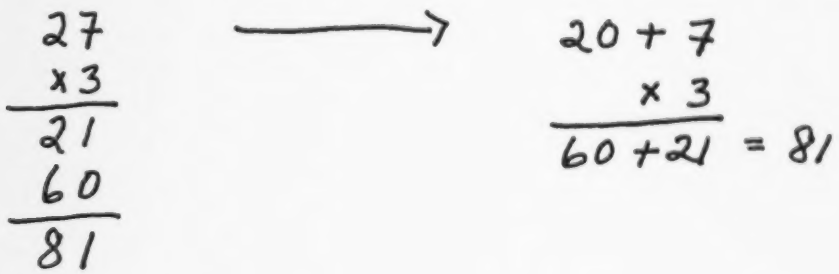


Figure 3.8

The teachers were astonished that we might think this was a good way to explain how to find 27 x 3 . This seemed like a step backwards! Rather than making the algorithm simpler, we were making it even more complicated, and certainly too long for students who already had problems with several steps. The teachers followed what we were doing mathematically and thought it was a neat trick, but they did not see it as a good approach to use with their students. We were surprised. Having seen the usefulness of owning a mental picture of what 27 x 3 was, why didn't this apply to the development of the multiplication algorithm? To us, the ability to break up 27 in this way, was an an important

property of numbers. The "Distributive Property", that one could break the number apart, multiply the parts separately, and recombine the result, was the essence of the standard algorithm. It seemed that this algorithm was a good place to illustrate the "sense-making" aspect of mathematics that students need to see. Still, as we explained what we saw as long-range benefits for the students, we had to appreciate the teachers concerns. Having visited their classes often, we could see how difficult it was for their students to listen to class discussions for more than a brief time or to explain their thinking in problems. What we were suggesting, without class discussion and thinking about the steps, could become just another procedure. Doubtful that such a discussion was possible, the teachers felt their efforts would be better spent on developing the concept of a product as repeated addition. The student could compute the product either by addition or by using the familiar algorithm.

One of the teachers' goals for this year had been that the students would learn how to apply the math calculations they were studying. So, we suggested giving the students a product, such as 27 x 3, and asking the students to make up a corresponding word problem. We thought that writing this problem in a journal, finding their answer, and then drawing a picture or diagram to show how they got their answer, might be a good way for the students to examine properties of multiplication. Perhaps it might encourage different approaches to finding a product. The teachers thought this idea would fit well with their class, that it would help the students to have to illustrate their thinking on paper, and that it might benefit their writing too.

The teachers in this group were dealing with a complex problem that did not depend solely on the focus one took in class or the style of teaching used. Lorel and I did take a different approach later on during the year when we ourselves taught several of these classes. But we did not find easy answers in trying to help this group of students. Our dilemma manifested itself in an area of concern that affected the upper grades in general, but was especially difficult for this particular group of students and teachers. Where does one place the emphasis in developing understanding and technical skill? Some might say that understanding must come first. Yet technical skill influences understanding and it is the interplay between these two abilities that is important. Knowing *what* an arithmetic operation means is a necessary part of knowing how to apply it, but given that, does one need to understand *why* a procedure works? For example, if one knows a way to compute the product, 23 x 50, and that this product is the total amount in 23 groups of 50 to a group, then one can quite easily determine how much a class will have if each of the 23 students in the class save 50 cents for a class party. What additional advantage was to be had in understanding why the multiplication algorithm worked? The benefits that we might see were not so clear to the teachers in the face of a pressing need to help their students reach grade-level skill in computation. Technical skill by itself may have its limitations, but its interaction with understanding becomes more important in the upper elementary grades as students encounter more advanced concepts. The teachers did not want their students to fall any further behind. At the same time they wanted their students to be exposed to some of the broader ideas about math that they themselves had seen during the summer. Several pointed out that frequently in the desire to help lower-achieving groups catch up

to their grade level classmates, the sole focus can become meaningless calculation. This was not what the teachers intended.

There are several important issues that need consideration in the kind of discussions about teaching math in which we were all involved (Heaton, R., Lampert, M., 1993). School faculty are very used to hearing 'experts' from a university tell them what is important to consider, in either math content or pedagogy, with only minimal regard for teachers' classroom knowledge as a group, and the individual teacher's own personal style of teaching. Although recently there has been some recognition of these factors, there is still the tendency, by both school and university faculties, to view in-service opportunities as ways to help schools implement a particular program. This tendency encourages teachers to doubt their own ability to design or analyze their classroom practice in math. Often this means that teachers are trying out suggestions that make very little sense to them or approaches in which they do not really believe. Our goal had been instead to develop a context in which teachers could consider in some depth the math they were teaching and what, and how, the students were learning. This requires more thought and effort than simply implementing a program designed by others. Not only does it require the teacher's deeper involvement in the mathematics itself, but it also requires the teacher's belief that her or his own ideas about learning and about math will be taken seriously, even in the face of conflicting points of view presented by experts in either math or in education. Working towards this kind of exchange takes time. One must not only accept that viewpoints on an issue may differ, one must be willing to try to see the situation from anothers' perspective, so that each one's contribution is a valued and necessary part of the dialogue.

Sharing the Classroom Experience

In November Angela asked me to teach her class.When I was considering what I would I do with her group of about 25 fourth grade students, I had asked Angela what topic she would like me to work on. She had said fractions because she thought this was always a difficult concept and she would like to see how I would begin. Angela had taken part in the summer math session on fractions, and she had been uneasy about the conceptual approach taken, one based on looking at parts of rectangular cakes drawn on graph paper. As our discussions progressed she would tell us that she was shaken by the experience and felt that she had little access to the math being done in the workshop. The discussions of math content seemed to take place at a level too far beyond her understanding. She had begun to doubt her ability to teach math, although she had considered herself to be a strong math teacher before the project began. From our grade-level discussions I had the sense that much of the work in Angela's class focused on practicing calculations, but that Angela was reconsidering the value to her students of having a more conceptual picture of what their calculations were about. I wanted to encourage this by doing something that would explore the students' thinking. I also wanted to develop the kind of activity that Angela would feel comfortable continuing with her class after my visit.

From my own experience with this age group, I felt that children were often rushed into techniques for adding and subtracting fractions before they really had any sense of

what the simple fractions were in different contexts. They often had little experience with how fractions related to each other, or even what adding fractions might mean. The ability to develop one's own diagrams to examine fractional concepts seemed especially important to me mathematically. Constructing our own pictures forces us to look at the role of the unit, the relational aspect of the numerator and denominator in a fraction, and to broaden our understanding of what operations with fractions mean. I decided to begin by simply looking at how the students would show $1/2, 1/4, 1/8$ on 4x4 geoboard squares drawn on paper, and to examine what kind of relationships among these fractions they might see from their pictures. This was an activity from the the book, *Seeing Fractions*, by Russell. I thought it was a very thoughtful book, one with many suggestions and questions for teachers to consider as they looked at their students' work. I also believed Angela might see the ideas in this book as reasonable. If she did, she could use the book as a reference in continuing the topic - my own work in her class would was to last for only a day or two at the most.

Angela organized her class by having the students do a set of four calculations listed on the board as soon as they came into the room. The students did these quietly by themselves, putting their work in a notebook, or journal. The journals were something that Angela had seen Deborah Ball use during the summer and she was adapting their use to fit her own working style and that of her students. After about five or ten minutes, Angela had the students exchange journals, and then called on a student to describe how to do the calculation. As she wrote the steps on the board, the other students corrected the work in the journals. She had recently added another step to this plan which she attributed to her work in the project. For each calculation, she asked the students to make up an appropriate story problem situation and they shared these with the class.

On the two days I taught the class, Angela began the class with a modified version of the plan just described. She wanted to give me more time, but she wanted to maintain what she felt was an important "settling in" procedure for the students. The first day I began by asking the students how they would show $1/2$ on the 4x4 geoboard square represented on the overhead. The students described in the air how they would cut the square down the middle horizontally, vertically, and then along a diagonal. At first they thought these were the only possible ways. So I drew the shape in Figure 3.9 below and asked if this showed $1/2$.

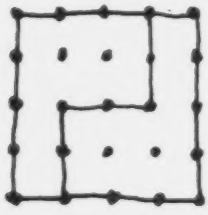


Figure 3.9

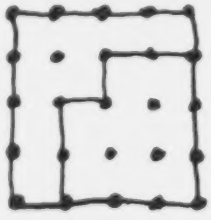


Figure 3.10

Tanya said the two pieces would fit on top of each other so it did show 1/2. When I asked the students if they thought other pictures were possible too, Stuart showed the shape in Figure 3.10 above, raising an important issue for the class to consider. Did the square need to be divided into two congruent shapes in order to show 1/2. At first the class felt Stuart must be wrong and several said so vehemently. But Stuart disagreed, showing that his two pieces were each made up of eight little squares. He claimed that just as with a cake, the pieces could be different shapes but hold the same amount, and so each could be 1/2. This argument seemed reasonable to the class. Continuing in this way, we went on to look at pictures of 1/4 as well and the students took turns coming up to the overhead to draw and explain their pictures. The students were quite involved in the discussion and eager to participate. Some like Stuart and Cathy showed a very good geometric sense as they worked to show why the pieces in their divisions of the square contained the same amount of space. We continued this the next day, adding in 1/8, drawing squares that showed all three fractions. We then began to look at writing fraction sentences such as $1/2 + 1/4 = 3/4$.

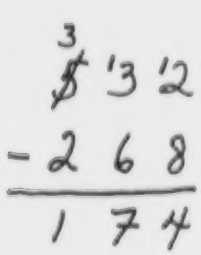
Angela and I discussed these lessons on the phone each night. She had been delighted to listen to her students' reasoning as they explained their pictures. She was especially pleased that the exercise had given students like Stuart and Cathy, who were very thoughtful about their work but did not do well in calculations, a good opportunity to be successful in the class discussions. She noted that the exercise encouraged discussion as there was lots of room for the students to develop and then compare their own representations of fractions within the given unit of a square. I suggested that by comparing their work, the students were drawn into examining each others work critically and in the process, expanded their own ideas about fractions. Angela pointed out that in Stuart's example showing 1/2, the question of equal amount of area versus equal shape had been raised precisely because the students did not intitially agree with Sterling. We both were interested to see how on the second day, some children were dividing the total number of squares in the geoboard unit square by 4 to find a picture of 1/4, whereas others where focusing on the geometric properties of the shapes they had drawn to justify their pictures. This could lead into developing ideas about equivalent fractions through the students' work.

As we continued to discuss the lessons, we both agreed that on the second day some students had seemed less involved. How should we modify the exercise, or what should the next steps be? Angela thought that the inattention of some students might be due to spending too much time on these few ideas and examples. The student's thinking about the concept of a fraction had been very good, but perhaps it was time to move on to look at methods for adding and subtracting fractions. I was more inclined to continue in the same vein but instead ask students to determine what kind of geoboard units they would need to illustrate other fractions. They could use their examples to compare fractions and develop a sense of what adding and subtracting fractions might mean. As we talked, Angela decided to continue the following day by having the students write in their journal as many fraction sums as they could see in one of their pictures showing $1/2, 1/4, 1/8$. Then she would move on to the algorithms for addition and subtraction of fractions. Angela's concerns about covering curriculum and keeping students engaged were important

to consider, so I suggested that as she continued on perhaps she could have her students look at simple fractions in other contexts such as time or money, and consider what addition and subtraction meant in these situations.

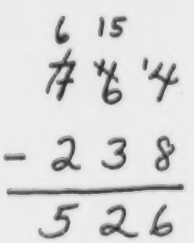
After these lessons Angela 'recommended me' to other teachers. I found the planning and discussion around the lessons I gave offered me and the teacher with whom I was working a very natural setting in which to try out and examine ideas about teaching math. I began to learn more about the mathematical issues that individual teachers were working on as well as their different approaches to teaching and learning. This helped me to develop my lessons around questions that would be relevant to the teachers plans and at the same time might offer new ideas or suggestions for us each to consider as math teachers. Also, I saw more clearly that the teachers were considering many issues of student growth over and above the math content which was my primary focus. More importantly, though, willingness to share the classroom experience with the teachers made us partners in considering how to teach mathematics. It didn't mean that we would have the same perspective or run into the same problems. Instead it meant that we could share more equally in a discussion about how to resolve these problems. In particular, critiquing what happened in a lesson became an integral part of teaching.

These experiences in the Washington classrooms provided many examples of children's work to share with my preservice students at the university. It helped me make real for my own preservice teachers issues that they would have otherwise been tempted to consider unimportant. For example, one day in class we examined the subtraction methods used by two fourth grade students. Their work is shown in Figures 3.11 and 3.12 below.



Example A

Figure 3.11



Example B

Figure 3.12

The first reaction of my preservice class was that both pieces of work were wrong. Then a couple of the students disagreed because they checked example B themselves and saw that the answer was right. They thought perhaps the student had got the right answer for the wrong reason. I asked these preservice teachers to consider what they thought the student's reasoning might be in each example.

Some of their comments were:

Regarding Example A

'The child borrowed from the wrong column - he should have just gone one column to the left to borrow.'

'You can see you have to borrow twice, maybe the child wanted to do that in one quick step.'

'Borrowing the 2 from 5 and putting a 1 for ten 10's over the 3 is okay. The problem is what to do with the left over 1 - it's not ten 1's, but ten 10's.'

Regarding Example B

'This method won't work all the time, will it?'

'You shouldn't borrow when you don't have to.'

'It looks like here you just borrow in each column right at the beginning. Then when get 15-3=12 in the middle column, you just write the 2 and mentally add back the 1 to the next column on the left. That's awfully complicated.'

After the students commented on the reasoning in these examples, I asked them to think about what they would do as teachers in working with these children. The discussion that followed developed around the use of standard algorithms and what understanding children need about subtraction. We concluded that children's thinking, even if it's wrong, can lead to learning; A number of students related their own frustration in elementary school with the procedures taught in subtraction and how they still used alternative methods. Healthy disagreement developed among the students as they considered their future roles in teaching. But through this and similar examples, they came to appreciate some of the demands that teaching would place on both their own mathematical understanding and their knowledge of how children learn. These discussions that seemed so valuable for my students convinced me of the need to integrate our preservice teachers' coursework. I began to think of ways to reorganize the mathematics preparation for elementary school teachers.

Relating Meeting Discussions to Classroom Practice

It was several months into the school year before I really appreciated the effects of a continuing conversation about teaching math. On a daily, or even a week-to-week, basis it was often difficult to see the progress in our discussions. Yet looking back one could trace the growth of ideas and the interaction between the grade level meetings and the classroom practice. Our work varied a great deal from one group to another but the following example illustrates the kind of exchange that gradually developed during the project. A group of third grade teachers had been working on the issue of borrowing in subtraction with three-digit numbers. They felt that their students' mistakes were largely due to lack of understanding or thought about the process being used. Marcy and Valerie

had the regular and high groups of students, while Diane had the low group. The grade level meetings with this group were always lively, and connections between math concepts and pedagogical questions arose naturally in these discussions. Marcy and Valerie were quite vocal, sharing lots of examples from their classes. Diane was quieter but very thoughtful. Early on as we had talked about subtraction, we had asked if they had ever used Base Ten blocks. These blocks can be used by teachers to demonstrate place value as shown in Figure 3.13 below.

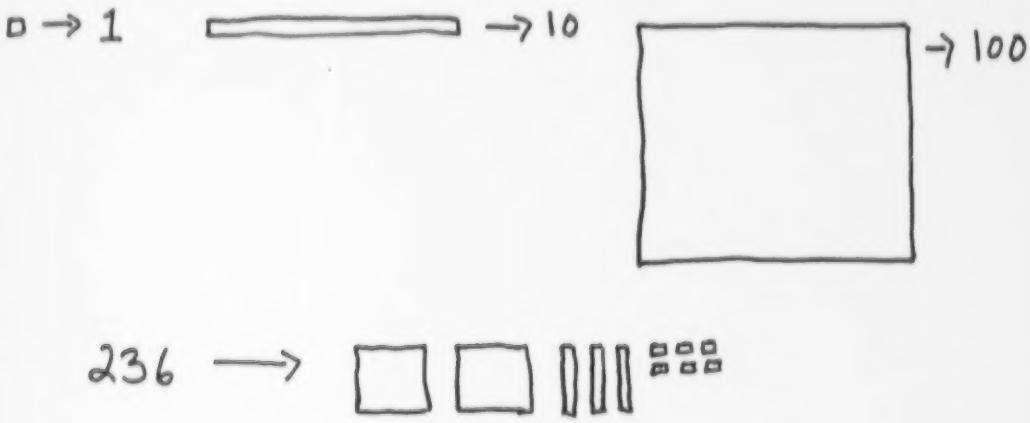


Figure 3.13

Marcy told us that she had but that it had seemed more confusing to the students. We asked her to show us what had happened with an example. Just as she did with the addition algorithm, Marcy modelled both numbers in working on the subtraction algorithm. For example in the problem (326 -154) she would represent both 326 and 154 using the blocks as shown in Figure 3.14 below.

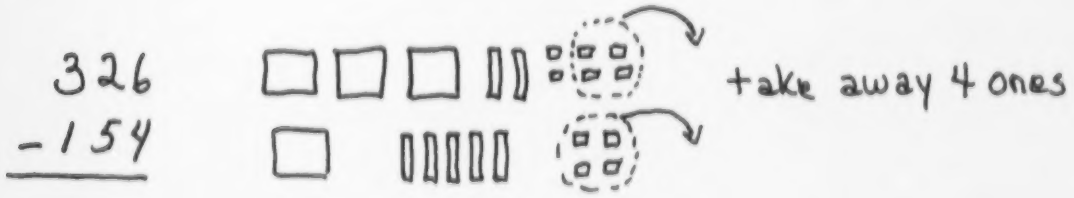


Figure 3.14

Since she and her students thought of subtraction as 'taking away', to demonstrate 6 ones 'take away' 4 ones, she would remove the circled blocks from each number representation. The students however became lost in what they were supposed to take away. Was Marcy actually taking away four ones in her demonstration, and if so, what was she taking them away from? The teachers considered this question and realized that the answer wasn't so clear! Their comments around this example had led to a good discussion of different concepts of subtraction, of how these concepts related to algorithms, and of the models one might use in demonstrations. In Marcy's example the teachers could see that representing only the number 326, and then physically removing from this model an amount of blocks corresponding to 154, was a better choice in thinking about subtraction as 'take away'.

Several weeks later Lorel saw how this discussion related to Marcy's work with her class. Marcy let a plastic disk represent a unit which she called a swamp person. Once students had ten disks, they put them together in a petrie dish and called this a boat. When Lorel arrived, Marcy had modelled a two-digit number on the overhead using boats and left-over swamp people. Marcy was tossing a die to determine the number she would subtract, or 'take away', and the students were describing how to subtract in terms of swamp people and boats and recording their answers on individual tally sheets. When Lorel and Marcy described the lesson in our meeting, Diane listened carefully and asked several questions. The next week as I dropped in on Diane's class, I saw her using the overhead and a set of unifix cubes to work on the same ideas. She was not accustomed to using manipulatives with her students and was a bit uneasy about how to manage this. The students clearly liked working with the cubes, but the amount of playing with the cubes worried Diane a bit. She felt that some of her students with behavior problems were too distracted by the possibilities for play that the cubes offered. But she did feel that many of the students seemed to be understanding more about the process of borrowing.

At our next meeting, we discussed the issue of playing with manipulatives versus using these objects to examine some math concepts. Then Marcy explained how she was now drawing pictures of swamp people, and the group considered how to pair up the pictures with recording the steps in the algorithm. Diane decided she would consider trying pictures too and on my next visit to her class she was in the midst of doing just that with her students. She would have students suggest the numbers for a subtraction problem, and while they explained what to do, she would carry out their suggestions on one side of the board with the pictures, and then walk to the other side, and illustrate what she had done with the digits in the problem. Her students were very attentive and pleased to show me how they used pictures to do subtraction. After a few board examples, the children worked some problems by themselves, drawing diagrams on their papers. When the class ended, Diane, who was used to simply telling the children what steps to follow in the algorithm, told me how delighted she was with this way of looking at subtraction. She said it made all the difference in the world. When I asked her *what* made the difference, she said 'having the children understand the steps they were using', and, "you'd never want to go back to the other way." During the summer, Diane appreciated the understanding she developed of some rules she had previously simply accepted, and she could see the usefulness of that

understanding. Now she wanted her students to have the same advantage. A few meetings later, Diane was to tell us how her students had sailed through the problems like (405-278) this year, thinking that borrowing with the zero in the tens place was 'no big deal', whereas in past years, this had been almost a whole new problem to consider. (See also Diane's own description of these events in Chapter 6: Personal Journeys)

Having a regular session to discuss the mathematics in their lessons and consider what their students were learning both allowed and encouraged these three teachers to examine their own thinking about math, as well as the goals they set for the students and ways to achieve these goals. Later on in the year as they looked at multiplication and division, Marcy brought an interesting question to our meeting. She had asked her students to tell her what they thought about the four arithmetic operations, a very general question. As the students talked, they began to compare what the operations did. There was a general conclusion that 'in addition and in multiplication, the numbers get bigger, but in subtraction and in division, they get smaller'. At first this had sounded reasonable, but then Marcy thought about multiplying by 0, $0 \times 4 < 4$. One of her students eventually raised this point in the class discussion. Now Marcy wondered about the statement in general. Dealing primarily with positive whole numbers, the conclusion might seem reasonable. As we looked at fractions and negative numbers though the teachers were surprised by the results. In that case, exactly what did these operations do, or what did they mean? We had looked at 3 x 5 earlier as the amount in three groups with five objects per group. Valerie brought this up, saying she didn't think it could explain 1/2 x 3/5. Marcy disagreed, saying that perhaps we could look at 1/2 as implying that we take 'half of a group'. Using a rectangle on graph paper, as shown in Figure 3.15 below, we saw Marcy's idea led to the same answer given by the algorithm for multiplication of fractions.

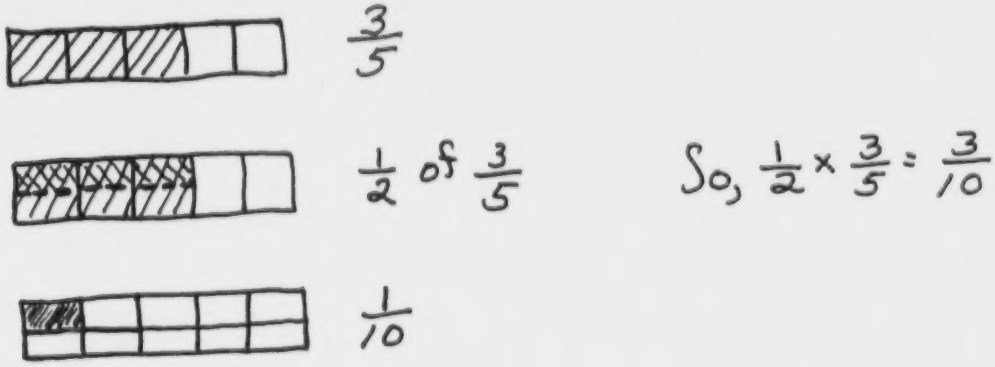


Figure 3.15

This event developed into an extended discussion about properties of multiplication, division, and fractions and the connections between these concepts. The teachers were especially interested in looking at the area model for multiplication on graph paper and

considering how this would allow them to examine these concepts in one context. Over the year the teachers used these ideas in different ways, sharing their class work in the meetings. Valerie, teaching the advanced third grade group, spent time on adding and subtracting fractions using a book Herb had written, *The Graph Paper Fractions Book*. Her students' computational skills were in general very good, but Valerie wanted to place more emphasis this year on developing the students' ability to understand the steps in their calculations and explain their reasoning as they solved word problems. She was very pleased towards the end of the year when students sought her help in problems for which they had an answer, but one that the children said 'didn't make sense'. Diane and Marcy spent more time on multiplication and division. Marcy would continue to ask very open-ended questions in her class. She enjoyed the uncertainty of the student response and frequently brought in comments that led to useful discussions. Towards the end of the first year when I visited Diane's class I found the students having a lively discussion about products and factors. As Diane offered the class a number like 36, the students suggested the dimensions of a rectangle Diane could draw on overhead graph paper that would consist of 36 squares. As she asked questions and represented the students' response on the overhead, she brought in very naturally related questions about fractions and division. Talking about the lesson afterwards, Diane told me that these questions on connections between the topics 'just popped out' now after the grade level discussions.

I was slowly beginning to see that the variety of activities we were involved in at the school, the meetings, the classroom visits, analyzing lessons, was developing into a very rich dialogue about math and about teaching math. One couldn't always predict the effect that a few comments in a meeting or after a class visit might have in the long run. But the opportunity for an on-going conversation around one's teaching and the interplay between one's own learning and the exchange of ideas with colleagues was a powerful combination. At the end of the second year Diane explained in her own words how this interaction was affecting her classroom practice. (See Chapter 6 for Diane's own story of her work in the project.)

> "I find that the "textbook approach" is a thing of the past. Rather than just opening the book in order to teach the next lesson and then proceeding to do a page of problems, I now have many resources I draw from, including several books, ideas from other teachers, and perhaps more importantly, the students themselves. ... I have learned to perhaps think of mathematical concepts at a higher level as we participated in classwork ourselves. ... Because I have given greater thought to the reasoning of some things (for example, borrowing in subtraction), I feel I now understand some of these concepts much better myself. ... I find myself asking different kinds of questions ...And, perhaps most importantly, I find math much more fun!"

A Sense of Shared Purpose

In January, a number of faculty from other elementary schools in Utah came to visit Washington School. Herb was developing Utah's proposal to the National Science Foundation's State Systemic Initiative. He hoped to extend the EMTTP project to six other

school/ university teams in Utah. The faculty and principals at these schools wanted to talk with us about the project and I had arranged for the visitors to drop in on classes, meet with groups of teachers to discuss their experience in the project, and then to meet with Carol, Herb and me for questions afterwards. The three of us were convinced that that we were making progress from the events described in the earlier sections of this chapter. But it was these meetings that helped us understand the teachers' sense of shared purpose and partnership in our work at Washington. The teachers emphasized to the visitors that the project was not a 'program' that one learned about and then adopted because it was approved by a principal. They described it rather as a process of thinking about how to teach math, one where you considered different approaches, and that there wasn't just one right method. Valerie explained that it was important to her that Carol didn't push her to adopt new ideas, but gave her time to consider her teaching in meetings and then to try out changes that she herself felt were important. The continued support during the school year and our willingness to look at both the mathematics and the classroom issues the teachers were facing was important to the teachers. They explained this had drawn them deeper into the project discussion. Louise, in comparing our work with that of another project, said that, "[in the other project] you knew you were doing the wrong thing mathematically " and that wasn't true for us. Initially I had thought these discussions with teams from other schools might be premature, but I was wrong. Thinking over how they would describe their work in the project for the visitors helped the teachers clarify their own understanding of this work and they were proud of their professional accomplishment. Carol was delighted to see the teachers' growing ability to articulate their ideas about teaching and learning math, and these discussions made it clear to us that school and university faculty were becoming equal partners in the project, each with different areas of expertise, but equal workers towards the goal of improving math education.

It is worthwhile considering again here why it does not seem better, or more effective, to simply develop a good program for teaching math, and then help teachers implement this with some support in the classroom for a period of time. One might develop a program that is mathematically sound and based on research about how students learn. Certainly a well-developed program is helpful as a resource, but it is only a start. Effective teaching also requires that one be able to take advantage of unexpected opportunities to bring together the main threads of the subject. This requires a fine-tuned sense of where the students currently are in their thinking in order to provide a means of accessing their potential. No program by itself can accomplish this. It is difficult to decide how to use students responses, or to know the limits of a model in examining a math concept, if one doesn't see how the concept grows across the curriculum. How can one know where to place one's emphasis in class without being aware of the thinking of the students? Finally, how and what one teaches must make sense in the light of one's own ideas about the subject, learning, and the role of a teacher. These necessary skills place much more emphasis on a teacher's knowledge and confidence, as well as a commitment to continued individual growth. We must recognize that not all teachers will want to reflect on their practice, and that even those who do will have different motivations for change and will go about making changes in different ways. Real change is difficult and teachers must see some benefit to themselves to make the continued effort necessary. Rather than a particular program to

implement, what is needed is a system of support that respects a teacher's own interests and teaching style, that offers insight into math and teaching so that each faculty member can grow and reach her or his own potential as a teacher.

CHAPTER 4

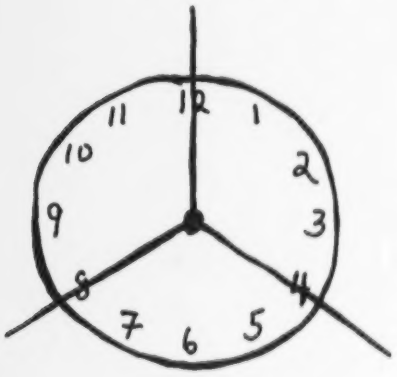
REFINING THE PARTNERSHIP

Over the first six to eight months of the project a sense of confidence and trust grew as those involved in the work learned to share their thinking about teaching mathematics. A partnership between school and university faculty had developed, one that encouraged a collegial exchange across the grades and that led to questions about long-range goals for the math curriculum. Addressing these questions however was not an easy or straightforward process and the discussions that resulted challenged our assumptions about teaching and learning elementary mathematics. In April 1993 the school faculty met to consider their work during the first year of the project. The teachers presented various viewpoints about what they had learned, the problems they encountered, and the changes they felt were necessary in the upcoming summer workshop. The discussion at this meeting and in the following months set a new direction for the partnership between school and university faculty. (February 1993 - August 1993)

The winter and early spring of 1993 was a time of slow but steady growth in the work of the EMTTP project. Lorel was on leave during the winter quarter, so I was visiting the school four or five days a week in order to maintain the level of contact with the faculty that we had established in the first part of the school year. Our conversations in the grade-level meetings about mathematics and teaching now seemed to be a more natural activity, one that was becoming better focused in several groups, and for all of us, a more effective process. As the dialogue about teaching math in the meetings became more fluent, so did classroom discussions in which students would be asked to explain their reasoning in solving math problems, or to share their thinking about math concepts. Listening to the students' responses led to many questions not only about the mathematics itself, but also about how students think about math, how they learn to apply their knowledge, and how their understanding grows and develops.

I was much more confident myself that our work together was benefitting us all and moving in a good direction. Earlier I had worried a great deal about the overall effect on the school faculty as they tried out new methods or ideas and, questioned assumptions about teaching math that had guided them in the past. Change isn't easy and people trying to make changes are very vulnerable. From interacting in the classroom with students and teachers, Lorel and I were learning a lot. But, at least at the beginning, it was not clear what kind of support we could offer that would benefit the faculty and which they themselves would regard as useful. Gradually, these concerns lessened as we came to understand and respect each other's goals and found effective ways to work together. Now as I walked through the school I knew many of the students from teaching in the classes. Both students and teachers were often eager to tell me what they had been doing in math that day, sometimes asking me to come and see what they had up on the board. Angela did just that one day in the early spring. She was feeling especially pleased with

her fourth-grade's work on the question,"How many minutes are there in two-thirds of an hour?". Following up on the joint work we had done in her classroom, she had been looking at the use of fractions in different contexts with her students. Now, asking a question about time, she explained that she chose the fraction $\frac{2}{3}$ to work with because she thought $\frac{1}{2}$ would be too easy and that her students might know how many minutes there are in $\frac{1}{4}$ or $\frac{3}{4}$ of an hour from other contexts. She wanted to see how they would use their knowledge of fractions. The two solutions shown in Figure 4.1 had been left on the blackboard. Angela explained that she was excited about the students growing ability to use fractions in other contexts, to explain their thinking, and to make comparisons between the approaches used. The class discussion in this example had led the students to think about connections between fractions and division. Angela felt that they were gaining a lot from listening to their peers talk about these concepts.



1 hour is 60 minutes

$$\begin{array}{r} 20 \\ 3\overline{)60} \\ \underline{60} \\ \end{array} \qquad \begin{array}{r} 20 \\ +20 \\ \hline 40 \end{array}$$

40 minutes

Figure 4.1

Angela's pleasure in her students' accomplishment and her consideration of what was valuable mathematically about their work was a focus shared by many of the teachers. Quite a few teachers were beginning to consider math topics other than arithmetic and to think about how math might be used in different curriculum areas. Many of the faculty had followed textbook guides in the past. Now as they examined their grade level goals, they began to critique and to supplement their textbook outlines. Individual teachers approached these changes and ideas very differently. But there was a keen sense of being a part of a group effort, one in which they could share their pleasure or their concerns and frustrations in their classroom work. At first it had seemed difficult to stay attuned to the different directions and questions of all of the teachers with whom we were working (approximately twenty). Now, watching the collegial exchange among the faculty grow, I appreciated the value of supporting this individual growth within a group context. Because the school faculty could take part in the project in a number of different ways, and because they were working on issues that personally interested them, there was a very high level of commitment. All but a few of the teachers were actively involved in the project work. The teachers were to tell us later that they spent far more time on the project than they'd originally planned. They attributed this to the personal freedom they found in pursuing

their own growth and to the support they received both from us and from their colleagues in their efforts.

Providing opportunities for individual growth and support was my primary concern in the beginning of the school year. Now, our grade level groups were developing a sharper identity, and I began to consider how these groups fit together as a faculty. This seemed an important step to me if the math program as a whole at the school was to benefit from the project work. The individual work, successes and failures, shared within the group, in meetings, at lunch, in the classrooms, provided a perfect background for refining one's own goals and thinking about change. Beyond that, it became an arena in which a school-wide sense of purpose and direction in mathematics could grow. The latter did not develop easily, or in a straightforward manner. But, with the encouragement and leadership of their principal Carol, examining the issues raised around the math curriculum was leading the faculty to new professional growth. In April, when our national advisors Deborah Ball and Deborah Schifter visited, they both noted a different energy and sense of purpose among the faculty from the previous summer and Deborah Schifter wrote the following words in her evaluation report.

> "There is a lot of positive energy at the school around mathematics education. Teachers are excited by what they and their students are doing. Many teachers feel that they have made significant changes in the way they teach mathematics and believe that those changes are reflected in students' greater confidence in and better attitudes toward mathematics."

Talking about Math Goals across the Grades

During each school year the faculty at Washington Elementary had three days set aside during which they did not teach. Instead they met to consider various school or curriculum issues. Within the district, these days were called 'Career Ladder Days' and were intended as professional development opportunities for school faculties. Attendance at these programs was not required, but those teachers who did attend were paid for their participation. During the EMTTP project, the faculty spent the mornings of these days on math issues. At the first Career Ladder Day in November Herb and I had led small math sessions. With quite a few of the faculty trying new approaches to teaching math, Herb proposed that we spend the second Career Ladder Day towards the end of January on assessment of learning in mathematics. This topic seemed to be a way to return again to the question of long-range math goals at each grade level and to examine how the math curriculum develops across the grades. Working with Carol, we came up with the following plan for the three hour meeting in the morning.

There would be three sessions and in each session, Herb would moderate a discussion of the K-3 teachers while I did the same with the 4-6 teachers. In the first session each grade would develop an rough outline of their math goals (topics; student attitudes; ways of thinking) and then the group as a whole would analyze how these outlines fit together. In the second session small groups of teachers at each grade level would pick a math

topic from their outline that they would teach during the next few months. Then they would examine in detail how this topic might best be developed. The last session would begin with a discussion of some material distributed to the faculty earlier on alternative assessment. These were: 1) The Sampler of Assessment Techniques from the Equals book, *Alternative Assessment in Mathematics*; 2) Bridges, a fourth grade assessment activity from the book, *Measuring Up*, put out by the Mathematical Sciences Education Board of the National Science Foundation. Following this discussion, the teachers would decide on both a traditional and alternative method for assessing the students learning on the topic they had been considering. Each teacher would bring to the third and last Career Ladder Day of the year, the results and the information gained from using these two methods of assessment.

The discussions during these two Career Ladder Days intrigued and occasionally disconcerted individual teachers. They compared assumptions about the math curriculum and the student work across several grades and, then discussed the issues that arose. In the 4-6 group, the outlines had indicated that all three grades planned to cover arithmetic operations with fractions. But did that mean they were all doing the same thing? Some of the teachers felt one had to go back each year to the very beginning of fractions, even with the 6th graders. If they were doing exactly the same topics, why was that necessary? If not, in what ways did these topics differ from one grade to another and how did they lead from one grade to the next? As the teachers talked they pointed out that even with whole numbers, the textbooks began with addition and subtraction all over again. Was this just meant to be a review? The sections moved faster, but the ideas were mostly the same with little new material. Often the students needed some of this work, but they were frequently bored and the review for many just provided more evidence that math was basically a dull, uninteresting subject.

Angela, Scott, and Jackie, a group of 4-5th grade teachers, had been working on a plan that they hoped would answer some of these questions. They offered to share their ideas with the group. Recognizing the need for review, but hoping to capture the students' interest early on in the year, they planned to work on whole number computation, and fraction and decimal concepts, in the context of geometry. They would begin with relationships between geometric shapes, but in working on area and perimeter of polygons, they would examine the above number concepts. In the second trimester, they would study arithmetic operations with fractions and decimals in some detail, using some of their geometry models. In the last trimester, they would work with percents, fractions, and decimals, perhaps using probability as a background. The 4th and 5th grade students were divided into a number of different ability groups. These teachers felt a common plan for the sequence of math topics as well as an outline of how topics would grow across the two grades was important in order that students have the chance to move between classes during the year depending on changes in performance.

As the group considered the teachers' plan and moved into a discussion of how fraction work might grow across the grades, some in the group began to reassess their own grade level goals. What conceptual ideas about fractions does a student need before operations

with fractions make sense? If all three grades (4th, 5th, 6th) cover addition of fractions, what will each grade focus on so that this concept builds across the grades? If one wants the students to be able to work flexibly with fractions and decimals, does it make sense to treat these topics separately? If the fourth grade spends more time on developing a conceptual understanding of fractions, will this help the fifth grade students develop a better grasp of how to work with fractions? What are reasonable benchmarks for progress in computational skills? One can learn a lot in this kind of exchange if the focus is on how the mathematics develops and effective ways to foster the students' learning. The 4-6 discussion seemed to moved along these lines, but the situation was different for a few of the teachers in the K-3 group.

As teachers, it is often disconcerting to learn that our students do not have a good grasp of a topic we have covered. When the second and third grade teachers compared their math outlines, Marcy was surprised to see multiplication listed on the second grade plan. She suggested that since the beginning third graders had little real understanding of borrowing in subtraction, perhaps the second grade should just spend more time on that topic, leaving multiplication till the next year. Sally and Suzanne were upset by this comment. They were already spending lots of time using manipulatives precisely in order to develop understanding of this concept. What 'understanding' was missing! Similar issues come up at all levels when looking across a sequence in which the courses build on each other. To discuss such questions effectively those involved must not only be open and flexible, they need a broad view of what learning in mathematics entails. Too often students' lack of knowledge or understanding is blamed on 'inadequate teaching at the lower level'. Teachers themselves, in deciding if their work has been effective, can be too harsh in their assessment unless they realize that learning about a math concept is an on-going, continual process. One of the benefits of having taken up some challenging math questions during the summer workshop was that the teachers themselves had experienced this process. Previously familiar and safe topics had began to take on new dimensions, sometimes causing doubt and anxiety at first, but making possible a deeper understanding of the topic. This experience was useful for the teachers now as both groups expressed their concerns and shared their goals for their students in several follow-up discussions. They examined in more depth what ideas about subtraction were important and discussed how these understandings might develop over the two grades.

Discussions about connections between the grades continued informally during the two months after the second Career Ladder Day. Lorel and I saw the assessment project become a useful focus for a number of grade level meetings as the teachers shared their students' work on the tasks selected. When we met as a whole group again in March for the last Career Ladder Day the school faculty shared their results on the traditional and alternative mathematics assessments they had tried in their classes. For the most part the traditional methods used were chapter tests or worksheets. The alternative methods included: students writing about how to solve a word problem or about what they knew best (or least) about a particular concept; drawing diagrams to show what a calculation like $(\frac{1}{3} + \frac{1}{2})$ means; asking questions such as, "What can I multiply together to get 24?". The teachers observed that they learned much more about the students' thinking from

the alternative assessment, and that this was useful to them in planning their lessons. They were delighted with this view into their students' thinking, finding the mistakes just as useful as the work that was correct. It did take more time though, and it was often harder to evaluate the work or to compare students' performances. Also the teachers were concerned that these methods might be more difficult for students with learning disabilities and/or language problems. The biggest drawback with the chapter tests and worksheets was that they seemed to rely on short, quick answers, or as the teachers said, "little bits and pieces". These methods did not tell much about depth of knowledge, but they could be useful as a quick check on specific skills. In the K-3 group the topic of assessment led to some questions about the practice of grouping students based solely on computational tests, an issue that would come up again in the second year of the project. Summarizing their experiences, the teachers in both groups concluded that a variety of assessment methods would provide the best assessment of a student's learning.

The day's discussion had focused on assessment, but sharing the results of the assessment projects encouraged us all to look at expectations for learning across the grades and to think again about a bigger picture of the elementary math curriculum. From these discussions Lorel became quite interested in the connections between the teachers' work on assessment and their thinking about mathematics and about teaching math, an interest which she decided to pursue in her doctoral thesis. (See Chapter 6 for Lorel's own story of her work in the project.)

Questions about Changing Images of Math

On a mid-year survey about the EMTTP project, we had asked the faculty to consider how their work in the project was affecting their thinking about teaching mathematics and their work in the classroom. The following comments written by the teachers illustrate some of the themes that the teachers were considering about learning math at this time.

> "I have also discovered that an environment which promotes debate and free expression is important to student exploration and learning."
>
> "I plan more time for exploration, discovering answers, discussion, thought questions, figuring and less time on practice/computing."
>
> "So much of what we do in math is interrelated. It's not just little units or chapters that can be taught alone. It all connects."
>
> "I am having to give what I am doing greater thought. The thinking process is a critical one to learn."
>
> "I find myself trying to really translate the processes more concretely and visually before introducing lessons to the students."

Encouraging students to talk about their mathematical ideas and the reasoning behind their work can produce intriguing and sometimes very useful results. Valerie had been working on beginning ideas about fractions with the advanced third grade students. Her

class surprised her towards the end of the lesson one day. Using the approach Herb had developed in his book, *The Graph Paper Fractions Book*, the students had just seen that $\frac{24}{36}$ is the same part of a whole as $\frac{2}{3}$ by drawing these amounts on two rectangular cakes of the same size. After comparing the diagrams, Valerie asked, "So, is $\frac{24}{36}$ the same as $\frac{2}{3}$?". A number in the class said no. Jack, one of her students, explained that he said no because one cake was divided into many more pieces which might be useful if you had to serve a lot of people. He continued on to say that what *was* the same, was that if you used one cake to feed only three people, then the amount eaten by two of them would be the same as the amount eaten by 24 when you shared the cake with 36 people. Jack was making a careful distinction that intrigued some of the students. His comment led others in the class to note that in the cake divided into thirds, each of these thirds would equal 12 pieces of the cake when divided into 36ths, an observation that could lead to a procedure for developing notions about equivalent fractions.

This observation could be an opportunity to make a useful relationship between equivalent fractions *sensible* in the children's context. Was this the time to pursue such a path? What sort of pictures would best illustrate the connections between the '3, 12, and 36'? Given another example, would the students be able to note similar relationships? A number of students still had difficulty determining the appropriate region of their cakes to illustrate a given fraction. How would they fare in such a discussion? Could one help the students describe their thinking so that a general method for finding equivalent fractions might develop? There is much more to consider when one's goal is to foster both students' conceptual understanding and their technical skill in doing mathematics! Often the answers to these question are not clear-cut, they vary from one situation to another, and they will depend on a combination of one's own mathematical and pedagogical knowledge. This uncertainty places great demands on a teacher. It can be far easier to simply provide the students with a useful rule than to attempt to connect this rule to the ideas presented in class. However, Valerie was an experienced teacher who valued the process of making sense in doing math and she wanted to pursue these questions. Talking with her after class, I thought again how important it is for our preservice teachers to examine these questions. They could learn a great deal by listening to practicing teachers consider the choices they might make in similar classroom situations.

Our meeting discussions were enriched by the closer attention that the teachers were paying to their students' thinking. "Why would she do that?", Sally asked after she explained to our group how one her students, Mary, had answered the question "34-18 = ?". The children were using beans and cups (1 bean = 1, 10 beans = 1 cup =10) and Mary had given 16 as the answer. When Sally had asked her to explain how she determined that, Mary had followed the steps shown in Figure 4.2 below. First she had emptied out two of her cups, taken away 18 of the 20 loose beans, and then combined the 2 left over beans with the remaining cup and beans to get 16. Mary's approach might seem strange at first, especially if one is expecting a demonstration of the traditional subtraction algorithm. However, writing down the steps Mary used in numerical form, 34=20+14, 20-18=2, 14+2=16, another reasonable approach became apparent. In symbols, Mary's

method is cumbersome, but using beans and cups, her method might be more efficient than that based on the algorithm.

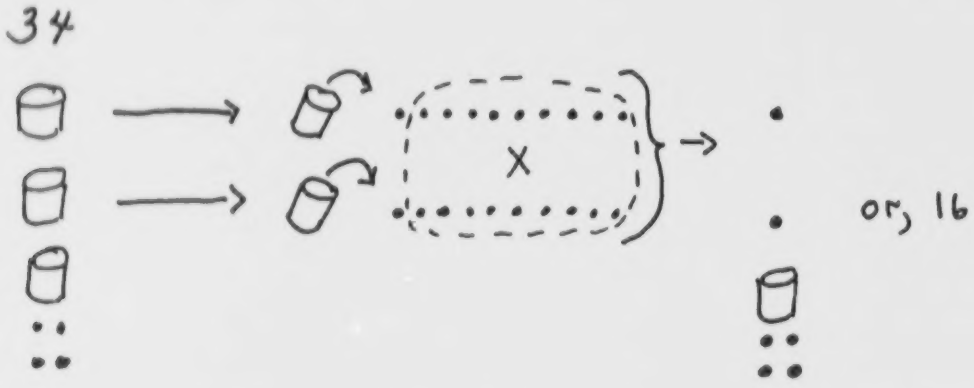


Figure 4.2

The teachers saw too that Mary's method demonstrated an efficient process to subtract mentally. (This example is related to one described earlier in Chapter 3 which also involved the flexible use of numbers.) But still, there were questions to consider. What might, or should, a second grade class do with these ideas? How is Mary's method related to the traditional algorithm? What will help students connect the work with diagrams/manipulatives to the abstract symbols used in representing the concepts being considered? Mary's approach or Jack's insight described above are valuable kernels of understanding, but how does one build on this knowledge effectively? This last question was important to consider for the future mathematical growth of the students.

Reflecting on Project Work After the First Year

In April 1993, towards the end of the first year of the project, Deborah Ball and Deborah Schifter returned to Washington Elementary for a three day visit. During this time Deborah Ball taught two classes while the teachers observed. Deborah Schifter visited several teachers in their classes and then debriefed with them afterwards. From their discussions with the teachers and their observations they noted that we were at a different stage in the project now and they made several valuable suggestions about the organization of the upcoming summer workshop.

Also at the end of this visit, the teachers were holding an overnight retreat at a local resort. Their agenda included time to discuss the math work during the year past and they had invited me to help them reflect on their efforts in the EMTTP project. Since Deborah Schifter was staying in Salt Lake City for another meeting, she was invited to the retreat as well.

At the retreat, as the session on the math project began, the teachers were seated at round tables in small groups. I suggested that they each recall their expectations when

the project began and compare these with their experiences over the past year. Then, working with their colleagues at the table, I asked them to discuss what impact they felt the project work had had on their students, on the school, and on themselves as teachers. After about fifteen minutes of small group discussion, the teachers were ready to share some of their table's comments with the group as a whole. Thinking back over the year, the teachers reported many positive benefits for their students resulting from the project work. These included:

- students were more willing to take risks, and were not as afraid of being wrong
- they could talk about their ideas, could think more on their own
- they saw math more connected to life around them.

The teachers believed that the emphasis on thinking and reasoning in math influenced studies in other areas too. They noted that in general the students asked more questions, and were more willing to think about the class material and then discuss their ideas. There was a more positive attitude towards math now in the school, both among students and among the faculty. The teachers said that they felt a strong sense of support in their efforts to improve math instruction. They liked the fact that the project allowed them to work at their own pace, that it was a collaboration based on looking for answers to questions about teaching math, rather than a set of ideas imposed on them. Again, the teachers pointed out that they had spent far more time working on the project than was originally intended. They attributed this to both the freedom to pursue their own development and to the fact that they were excited by the ideas encountered in their project work.

Listening to the discussion I could appreciate the teachers' pleasure in what they and their students had accomplished during the past year. What the faculty valued from their work varied from one teacher to another. But I had observed many of the effects that the teachers reported and I also knew the effort that went into bringing about that change. What relationship, I wondered, did the faculty see between their own learning and that of their students? Just as I was about to ask this question, Marcy proposed that we discuss plans for the upcoming summer workshop. Many in the room seemed interested in addressing this topic and I realized that they had a number of questions to raise. In fact, the issues that emerged in the ensuing discussion and that were examined in more detail in the next grade level meetings set a new and important direction for the partnership.

Questions about the daily schedule for the summer workshop and the teachers' stipend for their participation were raised first. Four weeks in August just before school starts is a big commitment and during the previous summer there had been a misunderstanding about the length of the workshop day. In the early stages of developing the proposal, the workshop was organized as a morning session involving a half-day commitment. However, the NSF daily stipend required participants to be active in a six hour session each day. As a result the initial plan was modified when the NSF indicated their interest in funding the project. In addition to the morning sessions on math content and pedagogical issues, workshop staff planned to work with groups of teachers in the afternoon. Together in small groups, they would design lessons related to the morning work and examine the materials and resources provided for the school by the project. This change in the time commitment

had not been made clear to the teachers before the start of the workshop. So, only a few had stayed in the afternoon. As project director, Herb, had agreed with a half-day schedule for the first summer because of the misunderstanding. But given the NSF regulations, this was not possible for the second summer. For the upcoming workshop we had explained to the teachers earlier in the year that there would be a choice: A half-day schedule with sessions in the morning for a reduced stipend ($40, $10 per hour), or the morning session with a homework requirement for a full stipend ($60). As the teachers thought ahead to the summer, they wanted to clarify exactly what their commitment would be, what amount of homework would be expected, and whether they could choose to do the homework periodically instead of every day. I discussed this briefly with the group.

Then the teachers' attention turned to the topics that the morning math sessions would examine. At first I thought that the group was uncertain about the main focus in these sessions, so I listed the topics we'd planned: Fractions; Geometry; Number Systems; and a new session, Integrating Math and Science, to replace the previous summer's topic of Estimation. But, I soon discovered that this was not what the faculty wanted to discuss! Marcy, reporting for her table, said that they didn't see any reason to be looking at approximating areas and relating this work to estimating the number π. They would rather spend time working on the kinds of activities that Deborah Ball used in her classes. Another teacher suggested that it would be really useful to have a file of all the many different activities that one could do in a class with the manipulatives that were now available, such as tangrams, unifix cubes, or base-ten blocks. Several teachers said that they had spent a lot of time learning the math, that this had been really useful. Now however they needed to focus more on how to use these ideas in the classroom, not on math topics that were unrelated to what they taught. A number of teachers seconded these comments and elaborated with their own ideas, but others disagreed. Both Lori and Valerie had been excited by their own learning and by the challenges they had met during the last summer workshop. Valerie explained that she felt this learning was a necessary part of the progress they'd seen with their students. Lori suggested that as a first grade teacher, it was the work on math together, even though it was above the level of her grade, that had helped her to develop a better sense of how topics in math connected across the grade levels. However, some of the teachers had had a different experience and had not seen their own learning grow. Angela said that because the math work during the first two weeks had been inaccessible to her the previous summer, she had gained very little from the discussion. She didn't know where to begin in asking a question, and she needed more direction from us as to how to approach the math we were considering. Angela wanted the kind of experience for herself that she saw her own students having during the past year at school.

I was both surprised and discouraged by many of these comments. Clearly there were some very different opinions about the value of the math work we had done together last summer. And, the connections that I thought we had made last summer between our math sessions and the K-6 classroom were not clear for quite a few teachers. I sensed the anxiety many felt about their commitment to the summer workshop in August. Yet, did the faculty really believe that learning more about math in the workshop was not

necessary, that instead it would be better to simply be given more classroom activities? When Deborah Schifter asked the teachers about this point, some said that they did need to learn more about math, but others were not so sure. This issue seemed very important to me. I felt it was precisely their own math work last summer that had drawn many of the faculty to a very different sense of what 'doing math' in a classroom involves. I had seen those summer experiences have a big impact during the year as teachers worked at asking 'good questions', thought about how to use students' responses effectively, and introduced less familiar topics in geometry or probability and statistics. But that was not everyone's understanding, and we needed to address the issues that the teachers were raising.

The discussion at the retreat was a turning point in our work together. Voicing their concerns about the summer workshop was an important step for the faculty. It was equally important for me to listen to these concerns and to appreciate the reasons behind them. Over the next month, in each grade level group I asked the teachers to help me understand their particular needs and goals for their summer math work. At the same time, using examples from our school year work together, I tried to clarify why I saw one's own learning about math in a broad sense to be such a critical factor in using good materials in the classroom, whether these materials are manipulatives, problems, or activities. In the resulting exchange we listened carefully to each other's viewpoints and began to share more equally the responsibility for stating our needs, evaluating our goals, and organizing the summer workshop. Working together, we developed a plan for a three week workshop, instead of a four week one. We would leave the practice teaching and debriefing, the fourth week activity in the previous summer, to take place in classroom visits during the next school year. As before, the first two weeks of the workshop were organized around three morning sessions. The first session would be an extended period for the teachers to continue their study of a weekly topic selected from: Fractions; Number Systems; Geometry; and the new topic, Integrating Math and Science. In this session, after a brief introduction of a topic, we would work together on math problems, alternating small group work with presenting questions to the group as a whole. The second session would be a time for teachers to examine resource materials, to continue work on math problems from the first session, or to organize a discussion around related classroom issues. During this time the teachers would pursue their own directions, choosing those activities that would best serve their interests. In the third session, those working on a particular topic would meet as a group. Then the teachers (taking turns) would lead the discussion they'd planned on classroom issues related to the group's topic.

As in the previous summer, Herb and I would lead the work in the first session. This year we were joined by a colleague, Hugo Rossi, who would lead the session Integrating Math and Science. The three of us would be available to consult with teachers in the second session when they pursued their own directions, but in the third session we would be participants. The organization of this last period around classroom issues was to be the teachers' responsibility. Just as in the last summer, during the third week of the workshop, Deborah Ball would give demonstration lessons with a briefing session both before and after her teaching. There would also be grade level meetings in the last week to discuss specific curriculum goals for the year, and Deborah Schifter would meet with small groups of

teachers to talk about the project work. The teachers would receive a stipend of $40/day for the morning sessions and there would be an optional daily homework assignment for an additional stipend of $20/day. The homework would involve math questions and related teaching issues during the first two weeks, and then essays on pedagogical issues that came up in the demonstration lessons during the third week. This plan seemed to have some flexibility and to satisfy both our's and the teachers' concerns. On the one hand it gave the attention that we felt necessary to learning about mathematics. But there was also time and context for the classroom issues that the teachers felt had been missing from the math work during the previous summer. As we finished the school year the teachers said that this summer they would have a better sense of what to ask for in the summer program, both from us and from themselves.

The Second Summer Workshop

Once the second summer workshop got underway, we were able to see a number of benefits from these discussions. Many of the teachers had attended the first summer with the expectations that they would be given detailed information about the math they were to learn, and then explicit instructions on how to do this math in their classes. After all, this has been the orientation of most teacher education programs in the United States. So, it was not surprising that the participants in the first summer workshop were unprepared to look on the experience as an opportunity to explore their own thinking, or to pursue their own questions about math content or pedagogy with the resources available. Now, however after a year of work in the project and with the discussions following the retreat, the returning teachers came back with a different sense of purpose about the summer work. They were more comfortable with open-ended discussions of math content or pedagogical issues. In their own classes they had begun to see that not knowing the answer and feeling confused was only a step in the learning process, perhaps a necessary one if one wanted to look at concepts deeply. This different understanding was a resource for those teachers who were attending the workshop for the first time, and it made the math work during the first two weeks more productive. This is shown in the following discussion with a group of K-3 teachers.

"What fraction is halfway between $\frac{1}{8}$ and $\frac{1}{9}$; halfway between $\frac{1}{8}$ and $\frac{1}{10}$? Is there a general picture you might draw that would show how to get a fraction that is halfway between two unit fractions?". This had been a homework question and the group was considering different approaches used. Sally and Suzanne decided to use rectangular cakes. They drew the picture in Figure 4.3 below but then weren't sure how to use this diagram to find an answer. Others in the group had run into the same problem and had decided to line up the two fractional parts as shown in Figure 4.4. They said the difference between the two fractions is one square. In that case, they went on to say, that by cutting each square in half, they would have 144 pieces. Then $\frac{1}{8}$ would be $\frac{18}{144}$, $\frac{1}{9}$ would be $\frac{16}{144}$ and

then, $\frac{17}{144}$ would be the number that is halfway between $\frac{1}{8}$ and $\frac{1}{9}$.

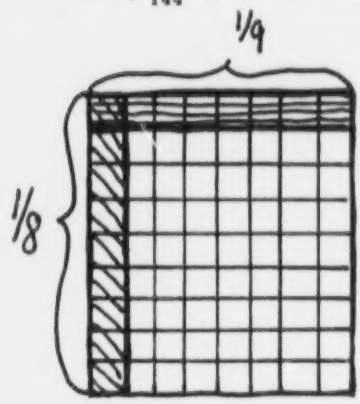


Figure 4.3

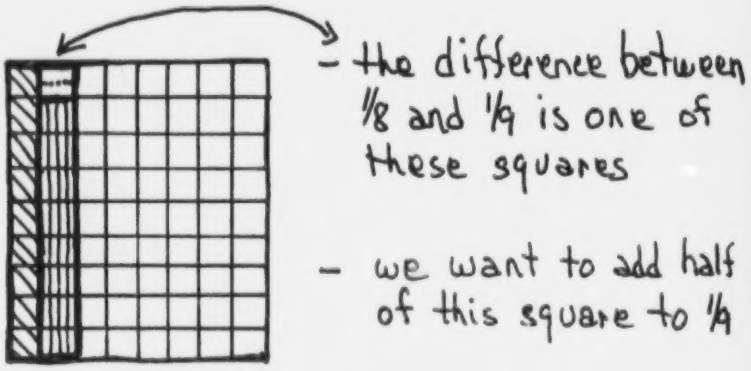


Figure 4.4

Someone in the group asked, "Is this the same as $\frac{8\frac{1}{2}}{72}$", and several teachers pointed out that some of their students wrote $\frac{1\frac{1}{2}}{4}$ as the number that was half-way between $\frac{1}{4}$ and $\frac{2}{4}$ instead of $\frac{3}{8}$. The picture suggested this was reasonable, but exactly what did the $\frac{1}{2}$ refer to in this case, $\frac{1}{2}$ of what? The resulting discussion led the group to develop conjectures about the general situation. In the dialogue, they returned to what they were coming to see as an important theme from our work in fractions. As Louise wrote at the end of the workshop, "During our study of fractions, the discussions always seemed to come around to the topic of 'units'–what size units were being used and compared to each other." This summer such issues made more sense to the teachers. The ability to examine these questions and to recognize their place in teaching fractions had evolved from the previous summer.

This second summer I saw the teachers doing math with much more confidence. The plan of having optional daily homework had turned out to be very useful. Quite a few of the teachers did this homework and they frequently stayed in the afternoons to work on the questions and investigations together. It gave me a very nice way to respond to individual work and at the same time it often led us into interesting discussions about the process of learning math the next day. The teachers were considering various strategies for learning and teaching in greater depth, as well as the connections they saw between their own learning and that of their students. Betty, a teacher new to the project, wrote about her thinking in the following words.

> "It was valuable to see so much demonstration of the process of bringing up what is confusing – or the questioning, different ways of coming up with the answer. ... People freely risking and expressing what part was confusing. It was support in just seeing the model of that process. ... All my life in school I had to draw pictures to understand what I was doing, but I always kind of did it so no one would notice. In other words, I was ashamed that I didn't know the necessary

algorithms ... It was wonderful to me that Anne encouraged pictures, extremely basic questioning, and group work in an adult group. ... I saw pictures were very reasonable and useful for not only me but many others."

The teachers' writings on both pedagogical and mathematical issues that arose in their math sessions indicated that their pictures of how one learns math were evolving and deepening. Planning ways to teach math was no longer quite so straightforward. John was tackling the question of how to balance conceptual understanding with technical skill. His struggle reminded me of Scott's question during a grade level meeting the past year. Scott had explained that he saw more and more the value in working towards conceptual understanding, but what percent of the time should be spent on understanding and what on skills? John wrote in his journal:

"The discussion generated by concentrating on why we invert and multiply a fraction to divide led to a variety of issues relating to math, and education in general. For instance, is it allowable for a teacher to tell a student to just "plug in" the formula to solve a posed problem? Is it always necessary for a student to know the conceptual foundation behind every equation when that foundation may be extremely time-consuming and very complicated; ...I believe that some students are not ready for some concepts. I believe that understanding may be gained by doing rather then by explanation. And I believe that the more a student understands the more freedom that student has to understand more."

Being responsible for the third period discussion each day, the teachers appreciated the part that their own expertise played in the project work and they were better able to relate their classroom work to the math content examined in the first period. This year Herb, Lorel and I had more understanding of the practical aspects of working with a class and this allowed us to participate more fully in the discussions led by the teachers. Lori commented on this change with the following words.

"There's a lot of processing to go through to make it work in your classroom. First, there's you as student and then there's you as teacher. This year they [workshop staff] helped us in doing the processing as teacher. They [workshop staff] didn't do that last year."

When we moved into the third week and Deborah Ball's demonstration lessons, the teachers' sense of responsibility grew as they shared in Deborah's inquiry into what had worked or not worked in her lessons. This summer the demonstration classes were larger. The students were drawn from the Washington School summer-school program, and they exhibited a greater range of abilities. Moreover, quite a few of the students changed from one day to the next, so Deborah had to adjust her plans for the lessons several times. The teachers saw this group of children as more representative of their own students, and as they watched the lessons there were many questions for them to consider. This year, as Deborah invited the teachers to help her set a direction for the next lesson, the teachers' discussion focused more on a combination of both math and pedagogical issues. Deborah incorporated some of the suggestions the teachers made into her classes. Together she and

the teachers discussed the results in the briefing sessions, and the teachers wrote about their thinking in their journals at night. As they considered these lessons, several teachers addressed the issue of responsibility for one's own learning and their goals to support students in becoming independent learners. After one lesson, Scott wrote the following in his journal.

> "I mean, if a teacher asks for the capital of Texas, wouldn't the reaction to such answers as Dallas and Houston have to be "Sorry, but no?" Then the elemental truth hit me that the answer to 3 x 4 is just as finite as the word Austin. A teacher could still accept or list all suggested responses and let students defend their answers. When one student pulls down the map or checks the atlas all students would see the correct response–and a method for finding it other than looking to me!"

In a discussion with Deborah Schifter, Lori and Betty commented on one of Deborah Ball's lessons in which a child had proposed an incorrect answer toward the end of a class. The answer had not been corrected and the second day the child returned with the correct answer. Lori noted, "Deborah didn't take it on herself to tutor the child. It was left as an investigation for the child." And Betty continued this theme saying,

> " So the misconception is almost like an opportunity for them to discover. If the teacher doesn't have to take responsibility to get the child to think "right", and if you can guide them so they aren't looking to you as the final authority but are looking to what is the truth about the matter, they're doing their own seeking, their own finding, and their own concluding, with your guidance."

By the end of the second summer workshop the sense of school and university faculty sharing the responsibility for project work had increased. In his efforts to expand the EMTTP project to other school districts, Herb had invited a number of teachers and principals to visit during the summer. In talking with these visitors, Carol, one of the teachers participating in the project from the beginning, had described her experience with words,

> " This is not a bag of tricks. The staff doesn't tell you what to do. But they help you think about the concepts you want to teach. In the classroom, you learn to listen to the kids for feedback on how they understand things. Then you continue, based on what you learn from them. This project has changed my teaching more than any other course I've ever taken."

Quite a few teachers would have agreed with the gist of what Carol was saying. Recognizing our different roles in the process of change and the fact that the project would end after one more year, I suggested that in the next school year we share the responsibility for organizing the grade level meetings. Lorel and I would attend every other meeting and the teachers would select one of their group to lead the remaining meetings. This would also give us more time to work with individual teachers in their classroom. The faculty liked this plan and we hoped to see some teacher-leaders develop who could continue to

organize the group's work after the project was over. As the workshop ended and we all moved on to think about classes for the next year, both school and university faculty had a better sense of their role and commitment to the project.

CHAPTER 5

NEW DIRECTIONS IN THE CONVERSATION

In the second year of the EMTTP the school faculty assumed more responsibility for the direction of the project work, sharing the role of leading grade level meetings and organizing Career Ladder Day sessions. This gave the university faculty time for more extended work with individual teachers. Here too the roles of both groups were changing when they shared the classroom work as colleagues. Through the dialogue about teaching and about mathematics, a community supporting the professional development of its members had developed. As the EMTTP project ended, the project participants discussed plans to maintain the conversation begun two years between school and university faculty. (September 1993 - May 1994)

In the middle of September 1993, I went to Washington School to meet with Carol Lubo so that we could check over our plans for the second year of the project work. School had been in session for about three weeks, and as I came into the building I met Scott. He asked me how long it would be before we were visiting classes again. I explained that we were scheduled to begin the following week and he told me that he had some things he'd like me to come and see soon. After my meeting with Carol, Scott saw me again on the stairs and asked if I had the time to come right then. As we walked back to Scott's room he told me that he had left his group of about twenty-four 5th grade students alone for a few minutes while they were trying to determine the area of three triangles drawn on graph paper. Scott had been using the material in Herb's book, *The Graph Paper Geometry Book*, in which each square on the graph paper is one square unit of area. The book begins by showing that since a right triangle is one-half of a rectangle, the area of a right triangle is one-half the area of the rectangle. Gradually this idea is extended to work with triangles in general, and then to finding areas of more complicated polygonal figures by subdividing these regions into simpler pieces.

When we walked into the classroom, I could see groups of students at desks around the room, all actively working on the problems. Scott had arranged an overhead in the front of the room and had the triangles drawn on transparencies. He gave the students a few more minutes and then he asked for the area of one of the triangles. Working without a formula for the area and seeing a number of ways to approach the problem, the students had come up with several different answers. Scott listed these down the side of the transparency. Then pointing to each one, he asked the students to raise their hand if they agreed with that answer. Beginning with the first suggested answer, he had the students come up to the overhead and explain how they had obtained their solution. When each student finished presenting their work, Scott asked the class whether they agreed or disagreed with the solution.

The students were quite involved in the discussion that followed this question. Although he moderated the discussion and occasionally helped clarify a comment or a question, he allowed the students themselves to compare their methods and to revise their

answers using the class's comments. The students clearly felt both interested in and able to determine the validity of a solution. Talking in the few minutes between lessons, Scott told me how pleased he was with the way his class was going. During the second summer workshop he had been very intrigued by what he called Deborah Ball's "even keel" manner, and contrasted it to teaching styles in which a teacher offers immediate praise or feedback for students' answers. He saw that she accepted all answers and then expected the students, individually and as a class, to examine these responses seriously by sharing their reasoning with their classmates. A month earlier he had described his reaction to this teaching style in the following words.

> "I am again impressed (and a little bit confused) concerning Debra's [Deborah Ball's] low-key style of responding to the students. The behavior modification techniques of praising correct response is not seen in her teaching ...is this style of interaction with students superior to Skinner's in that it enables students to take more responsibility for their learning and develop intrinsic gratification? I am leaning toward the latter [Deborah Ball's approach] as I watch Debra and the students. The learning,the content, seems to come more from the students than the teacher."

Scott had a new style of classroom work in mind as he began the second year of the project. Once back in school, during the first few weeks, he had worked to establish the social climate necessary for students to discuss each other's solutions to math problems. This was a very important step for the kind of discussions he wanted to develop and he also felt this work in itself was valuable for the students' learning. Scott was delighted to see the students more excited about their work. He felt the students liked mathematics more as a subject and thought his new approach was allowing more students to take part in the classroom work.

Scott invited me to come back the following week to see the class's progress in their work on area. As I walked away I sensed that the manner of our work together was changing. In the previous year Scott had mulled over issues about teaching math that were raised in the grade level meetings. Although I had visited Scott's class several times that first year, it wasn't until the Spring that we had shared much classroom interaction. Since then Scott had given considerable attention to his own thinking about math and to the goals he held for his students' learning. Now he wanted to try a new direction in his teaching. Moreover, he wanted to work together as colleagues in examining the effects of the changes he envisioned on his students' learning. In the next few weeks, I recognized this change of direction in my work with several teachers and I was glad that this year Lorel and I would be spending more time working with individual teachers in the classroom. Carol Lubo had told me that the faculty had returned to school quite excited about their work in math. In planning classroom visits with the teachers during the first weeks of school, many of them had asked her to come during their math lessons. Carol felt they had discovered an openness, a sense of pleasure in discovery, and room for creativeness in math that they hadn't previously seen in the subject.

There were three new teachers joining the faculty this year and there were also a few changes in the grade level groups. For example, the previous year the 4-5th grade teachers

had met in two math groups depending on the ability level of the students they were teaching. This year the teachers of all of the 4-5th math groups planned to meet together because they felt it was important to include those working with all ability levels in their math discussion. There would still be a total of six groups of teachers meeting however, because the kindergarten teachers wanted to have meetings for their grade this year as well. Just as in the first year, each group would meet every two weeks. Now, however, Lorel and I would attend only every other meeting. I was uncertain how effective this plan would be. I wondered whether we would be able to keep abreast of the current concerns of each group. This knowledge had been very important to our work in the previous year. Also, would the teachers need help in focusing their discussions in the grade level meetings? During the first year, it had taken some planning to lead these discussions and we had found it necessary to send memos summarizing the current issues. What steps would the teachers take in setting a direction for their meetings and how could we support the discussions this year, rather than lead them?

In thinking about these questions, I remembered that the writing the teachers had done for homework during the summer workshop had been a very effective means for the faculty to reflect on teaching and learning math. Perhaps a writing project during the school year would also be useful and it might provide some continuity for the grade level discussions. I decided to propose the following three topics during the school year, one each trimester, for the faculty to consider.

> Report 1: Describe one of your math lessons. Report on the dialogue in the lesson, and discuss the students' thinking involved in the dialogue.
>
> Report 2: Select one of your math lessons that influenced your thinking about teaching math. Describe the lesson briefly and discuss what event(s) in the lesson were important to you and why.
>
> Report 3: Describe in what way your involvement in the EMTTP Project has affected you and your students in the process of doing mathematics. What advice would you give to faculty at other schools who would like to engage in similar projects?

Teachers Refining Their Classroom Goals

During the second summer workshop the participants had examined with Deborah Ball the questions she was asking herself about her teaching in the demonstration lessons. This group dialogue along with the work of the previous year meant that many of the teachers began the second year with a more analytical outlook on their teaching and their students' learning. The faculty were considering a variety of mathematical and/or pedagogical issues. When I returned to Scott's class a few weeks after the visit described above, I could see that in addition to examining how his teaching style might foster student initiative in the learning process, he was considering again some important questions he

had about math goals. At the end of the first summer workshop, Scott had written the following excerpt in his journal.

> "I find myself intrigued and confused by the line of thought that students are better off being taught to think and reason, than taught to memorize formulas and procedures. When first made aware of this type of thinking it made tremendous sense to me. ... However, the pendulum has (is) slowly moving back toward middle ground. I find there are students -majority left-brained??-who function better??(or is it habit) with rules and procedures to use as a foundation, then base their reasoning and problem-solving from there."

We had talked about these questions during the first year, and one day in the Spring of 1993, I had asked Scott if he would like me to teach his class for a day or two. I thought this would give us a situation in which we could examine some of the questions he was considering. He said yes. He was working on fractions, and he would like to see me do something that combined fractions with estimation. I visited his class a few days before I would teach. I saw that the students were working with mixed numbers, having already covered the algorithms for addition, subtraction, and multiplication of fractions. There were about 12 students in the class, all above grade level in math. Four were very quiet girls, and the rest, quite boisterous boys. As in working with Angela's class earlier in that first year, I had wanted to look at the students' understanding of the fraction concept. Since Scott had asked me to include estimation, I decided to use an activity suggested in the NCTM Addenda book, *Number Sense, Grades 5-8.* In this activity students work with five or six fractions, all between zero and one, first sorting them into two groups, those less than 1/2, and those bigger than 1/2. Then they arrange the fractions in order from smallest to largest. I had tried this exercise unsuccessfully in the beginning of the school year with a sixth grade class. However, I liked the focus of the exercise a great deal. This time I had the advantage that Scott's students had been working with fractions very recently. I also had some better ideas about introducing estimation with fractions in several situations that I thought the students would appreciate. So, I decided to try the exercise again.

We spent a day and a half looking at the exercise, having the students work in groups and then share their thinking with the class. I had found it necessary to give students some guidelines about listening to each other's explanations, but on the whole the discussion was very good, with students using some of the following reasoning:

> "$4/7 > 1/2$ since half of 7 is $3\frac{1}{2}$ and $4 > 3\frac{1}{2}$"

> "$9/10 > 7/9$ since $9/10$ is $1/10$ from 1, $7/9$ is $2/9$ from 1, and $1/10 < 1/9 < 2/9$."

At one point, a student, Ed, had suggested that $1/4 > 2/5$ because in the denominators, $5 > 4$. This led another student, Rebecca, into trying to explain that the reasoning, "the fraction with the bigger denominator is the smaller number", only worked in the case when the two fractions have the same numerator. All but two of the students were quite involved in these question, and I was delighted to see the girls taking an active part in the discussion.

When Scott and I talked at night about the class, he commented that we had learned a lot about the children's concept of a fraction. He also noted that some who were able to use the algorithms well were not so clear on the relationship between numerator and denominator. He was surprised that few had used common denominators to make their comparisons and asked if I had avoided this on purpose. I hadn't, but with seven fractions to compare, I had expected that estimating would seem an easier approach, at least with most of the fractions. I also hoped this would give us some insight into the students' thinking about the fraction concept. That made sense to Scott and he said he could see a number of ways to continue with this work in his class. At the next 4-5th grade level meeting Scott had talked about this lesson, explaining to the group some of the questions he was considering about learning algorithms and conceptual thinking. Scott was seeing a need for his students to have both understanding and technical skill in math. How does one achieve both? Must one come before the other? Are these conflicting goals? In the reality of a classroom, these are not easy questions to answer. But they are questions that teachers and students at all levels must consider. Scott was taking up these questions again in the second year of the project as he began work on multiplication facts with his students.

As described at the beginning of this chapter, Scott had started the second year with geometry and was intending to use this context to review operations with whole numbers. After summer vacation, students at this grade level often need to work on the multiplication facts. Scott had determined that this was the case with his class and we talked about ways to go about this review. Scott didn't want to just drill facts, but for the more advanced computations they would be doing this year he did want the students to have fairly quick recall of multiplication facts. Coming into the classroom, I saw Scott showing a multiplication table (0-10) on an overhead and discussing with his students that there were 121 facts in the table. While the students groaned, Scott reminded them that the class would use these facts in lots of ways this year and that they needed to know them quite well. Then he asked the students which they thought were easy to memorize and why. The students quickly suggested the facts with 0, 1, 2, 5, 10 were easy, and offered their reasons. Scott wiped out the corresponding rows and columns of the 'easy facts' since the students didn't need to work on them. Considering why he wiped out both the row and the column corresponding to 2 led the class to see that in fact they could wipe out all of the facts below the diagonal since (a x b) = (b x a). Together they had reduced their work from 121 facts to 23 'hard facts' to practice. By giving the students the lead in deciding where their study efforts would be best placed and enlisting their help in reducing the work, Scott turned what is often a drill and practice exercise into an opportunity for the students to use their understanding of the math to simplify their work on developing a necessary technical skill. Scott assigned the diagonal facts for the students to work on for the next day. Then the class returned to their work on finding areas of shaded regions in the plane such as in Figure 5.1 below. Now with the square unit of area only indicated by the tick marks on the boundaries, in contrast to their earlier work with regions drawn on graph paper, the students had to draw in or picture mentally the grid lines they had previously used. Scott used the students' discussion around their solutions to bring up the difference between linear and square units, to consider right, acute, and obtuse angles,

and to introduce the symbol commonly used to indicate right angles. Determining the area of the given regions naturally involved computing various products. This work gave the students a context in which they could examine the operation of multiplication and appreciate the usefulness of a quick recall of multiplication facts.

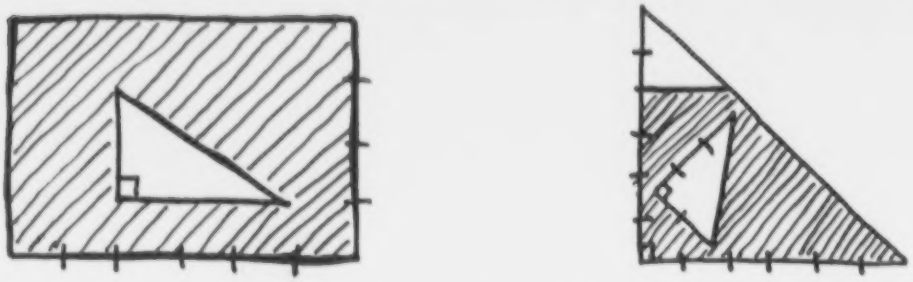


Figure 5.1

That night on the phone, Scott said he was pleased with the students' work on the multiplication table. He planned to use a game situation the next day to test their knowledge of the diagonal facts. The game would involve speed of recall and Scott explained that while he didn't want to stress speed too much, having some time factor actually was an incentive for many of the students. He also felt his students were developing a much better understanding of area this year. Now they knew not only a formula for the area of a triangle, but also *what* was being measured by that formula. Even better, they had a way to approach finding areas of regions other than rectangles or triangles. Our discussions continued over the year in classroom visits and in grade level meetings. Scott thought seriously about the effect of fostering student initiative on the students' learning in math and in other areas. In his Teacher Reports, he wrote about this issue and examined some of the connections he saw between developing students' conceptual understanding and technical skill. In his last report he described the benefits he saw from his work in the following words. (See Ch 6 for Scott's own story of his work in the project.)

> "Students seem to regard themselves as more competent mathematically ... Emphasis has shifted away from a right or wrong, black or white, "all or nothing" perspective. Students have more opportunity to regard themselves as contributors to a growth process ...to view their partial understanding as normal. Relative (or casual) to this shift in student attitude is a reformation of teacher presentation. Instead of "This is the way you do it, class", I find myself saying, "See what methods you can devise to solve this problem." The latter instruction allows considerably more opportunity for challenge, comparison, esteem, fun, success, and original thinking."

Along with Scott, other teachers too were considering how their teaching style influenced the interaction and the learning in their classrooms. One of these was Carol, a teacher at another school in the district who had participated in the project from the beginning. Although Carol did not have a group of colleagues at her school with whom she could share her work, during the first year of the project Herb had visited her once

every two weeks. This year I was doing the same thing and Carol had been able to arrange her schedule so that we would have the half hour after her math lesson, which lasted 75 minutes, to discuss the events in her class. Carol had switched from a fourth grade to a second grade this year and, as the faculty at her school had decided to stop ability grouping, Carol had quite a range of students in her class of twenty-four. In contrast to Washington Elementary, Carol's school was located in a middle-income neighborhood and she had regular parent support in her class.

Like Scott, Carol had been eager to begin this year. In setting up a time for me to visit her class, Carol had told me that she wanted to give a broader focus to the mathematics in her class this year. She hoped to work on the required concepts and skills through investigations related to the children's world and a careful dialogue about their findings. When I arrived for my first visit, she introduced me to her class and asked me to join their circle around the large calendar displayed on the blackboard. The day was October 7th and the squares on the calendar corresponding to the dates October 1st through October 6th contained either a purple circle or a yellow square, arranged in a pattern. "What should we put on today's square?" Carol asked. One child gave an answer and as Carol put up the suggested figure, she asked the group if it would fit and how they could describe the pattern. Lots of hands were raised. The answers given described not only the color patterns and the shape patterns, but translated the pattern to other situations by describing the circle as a cookie and the square as a sandwich. Carol asked them to extend the pattern several days and then to predict what would fall on dates a week or so ahead, and to explain how they made their prediction. She listened carefully to their responses, asking questions and inviting the other students to comment on the answer given. The student of the day was asked to make up a word problem having the day of the month, 7, as the answer. Carol posted this problem beside the calendar, then turned to a long strip of white paper taped horizontally along the wall just below the calendar.

The numbers 1, 2, 3,..., 28 were written in order from left to right with lots of room on the tape to continue. This tape was a timeline indicating the number of days school had been in session. Reporting that they'd been in school now for 29 days, the students talked with Carol about writing 29 on the paper strip with the '2' in red because it showed how many tens there were in 29, and the '9' in black to show the leftover ones. After this Carol invited a student to add a penny to their collection of dimes, nickels and pennies that also showed the number of days in school, making whatever exchanges in the coins were indicated. Here the students were asked to predict when various exchanges would be needed and their responses were tied in to the timeline and the calendar. For the rest of the class, the students sorted and then displayed on a large floor graph, a collection of apples that they had brought to class. They had found 30 red apples, 3 green ones, and 12 of mixed color. To get the total number of apples, some counted the apples in all three piles starting with 1, others counted on from the number known to be in a given pile, and one child verbally explained his answer of 45 this way: " 30+10=40, then 40+3=43, and 43+1+1=45" which Carol wrote on the board.

As we sat down together to talk after the class, Carol said that she felt ready to

work on a different model of teaching math, one that she had seen from the students' perspective in her math sessions during the summer and one that she had watched being played out in Deborah Ball's demonstration lessons. In her journal entries she had written about a "rapport" in the math sessions that "wasn't just based on a questioning technique but a whole way of interaction". She felt this was based on treating students and their ideas with respect, and "non-intrusive" questioning that encouraged students themselves to examine ideas in greater depth. This helped the teacher-participants to find the answers they were seeking, and to look at concepts in several different ways. Thinking about her own classroom however, Carol was uncertain about how to begin and what to expect from her students. Responding to a discussion led by Deborah Ball and Deborah Schifter on the role of the teacher in classroom discourse, Carol had written about her concerns in these words.

> "What if none of the students ever begin to know how to tackle this problem, what if they all just sit there? As I ruminated about this I realized this fear of the unknown was what held me back from this approach [to teaching]. It's safe to have a plan in mind and to follow it. Do I have the courage to let things evolve? Do I have the mathematical background to direct? Will we get too sidetracked? What do I do with the students who don't engage?

Through the daily math calendar discussion with her class Carol had found a good way to work on some of the changes she wanted to make. This was a new activity for her, one she had thought might foster the kind of dialogue in math that she wanted to develop this year. I told Carol that I saw this introduction to the day's math class giving the students some wonderful opportunities to think about important number concepts and how numbers could be used in situations related to their own experiences. Carol agreed. She said that in responding to her 'What if ...' questions about the extensions in calendar patterns and the timeline of days in school, and in looking at some related measurement activities that changed daily, the children were naturally considering many aspects of addition and subtraction. It would be important for the students to develop skill at these operations this year and Carol was mulling over how she would approach these topics. She asked what I thought of the method described above to find the total number of apples. I explained that to me this showed good number sense and an understanding of place value. Computing mentally, it was very efficient. This comment lead us into a discussion over the next several visits. This discussion centered on working with numbers flexibly, why this was useful, and what her students' observations showed about their understanding of numbers.

Working towards the broader mathematical focus that she desired, Carol was noting that she had to give a lot of thought now to exactly what her math goals were and to how the concepts built on one another over the year. She had decided that recognizing number patterns and interpreting place value were important aspects of having good number sense. She also thought that having the students work with numbers in other bases using manipulatives would be a good way to deepen the children's understanding of the process of grouping in Base Ten. She had a number of different scenarios in which these other bases developed and one day in the winter I watched the class working in Base Five. Here 1 bean represented an astronaut, and 5 beans in a cup made a spaceship. Working with ma-

nipulatives, the class would add astronauts to their pile, regrouping (making spaceships) when necessary. When they reported the results to Carol in terms of how many spaceships and how many astronauts they had, say 2 spaceships, 4 astronauts, Carol recorded the symbols 2,4 on the board in the columns labeled 'spaceships', 'astronauts' respectively. At one point I suggested to Carol that she ask how many astronauts there would be if a report of '32' had been made. Carol thought about it, but decided instead to look at patterns in the Base Five numerals written in order and compare this to what they had seen another day when they looked at Base Four. A lively discussion followed in which some students merely described patterns, but a number talked about the effect of the group size on these patterns, making predictions about how the numerals would continue.

Thinking over the children's observations afterwards, we considered how to relate their work now to what Carol was planning to work on soon in Base 10. I asked her what she thought about the question, 'How many astronauts does 32 represent?'. As she began to explain why she thought it might be too confusing, with the 3 standing for spaceships and the 2 for astronauts, she stopped and said, 'but that's the whole point of bases, isn't it'. She went on to note that for children using the Base 10 number 53, knowing that 5 was in the tens place and 3 in the ones place is not the same thing as realizing that this means 53=50+3. Carol thought about place value and its role in number sense and computations over the next several visits. When she moved into addition and subtraction in Base 10 she found that many students wanted to work with the 'tens' first. She was a bit concerned about this initially, but she also saw the value in the students' thinking and was more confident in letting their ideas lead the way for awhile. Eventually the students did consider the common algorithms, and at the end of the year, Carol was delighted when her class did very well on standard computational tests. More than that though, they had a different picture of what math was. Describing this in her third Teacher Report she wrote,

> "They think of math as something that has a pattern that can be figured out. They enjoy explaining their strategies and observations. They don't see math as a series of steps to be memorized, rather they see it as a logical process that can be explained.

Carol continued on to write about her learning through her work in the project (See Ch 6 for Carol's own story) in the following words,

> "I finally figured out that I needed to experience making sense out of mathematical concepts on my level in order to understand what processes the students needed in order to make sense on their level."

Scott, Carol, and several other teachers were working to change the pattern of dialogue and interaction in their classrooms. They saw many learning benefits deriving from the students' more active role and they wanted to organize their discussions around the students' thinking. But valuing this kind of student interaction and thinking was not an end in itself. They also valued mathematical growth. In determining what this might entail, both at their grade level and in the long run, these teachers were drawn into deeper questions and involvement in the mathematics itself. The teachers appreciated the need to

examine the mathematical issues as well as the questions of teaching. I believe this appreciation was in no small measure due to the teachers' own intensive work in mathematics during the summer workshop.

It was a very valuable experience for me to be working with these teachers for longer periods this second year. Many of the questions and problems that the teachers were considering are not unique to the elementary school situation. Although the answers may differ and the role of teacher and student may change depending on the level, the questions about how students learn and how they come to use their knowledge effectively concern all teachers. It was this latter question that I had uppermost in mind when I thought about our preservice courses in mathematics. Prospective elementary teachers need a knowledge of mathematics that helps them to guide the learning of others, the kind of knowledge that moves beyond simply being able to do mathematics for one's own purposes. Thinking about ways to help our preservice students develop this in-depth knowledge of math and of teaching drew me into a discussion with a new colleague, Janine Remillard, who had joined the Department of Educational Studies. Together we began work on a proposal for a pilot program to integrate the mathematics content and methods courses at the university.

A Community Supporting Professional Growth

Several factors differentiated our work in the second year from the beginning of the project. By now a broad context had been established for this conversation about learning and teaching math. There was a measure of confidence in the partnership that had grown during the first year. A climate had developed that encouraged reflection on one's own teaching and helped the faculty value their own professional development. While there were two or three teachers who were only minimally involved in the project work, most of the faculty were active participants. As several teachers said, we were all "in a different place this year."

In the first year the faculty had approached their classes with some new ideas about math as a subject and about ways to teach math. These ideas had seemed valuable to the teachers as they had worked together in groups or watched demonstration lessons in the summer workshop. Most judged these new ways to be of enough value that they wanted to incorporate these experiences into their classroom work. But determining how to do this is a slow and gradual process, as both Carol and Scott explain in their own stories (Chapter 6). For example, we may believe, in a general sort of way, that students drawing diagrams to explain their thinking is useful. However, until we have experimented with this kind of work in our lessons, have discovered the benefits and the problems of this activity, and have considered its affect on our students' knowledge, drawing diagrams will remain someone else's useful approach to looking at math concepts. It is only through such experimentation that we can determine for ourselves how drawing diagrams can be useful to *us in our teaching* and can support our own learning goals for students. Along the way there will be many questions, doubts, and uncertainty.

Because the entire faculty at Washington Elementary was involved in the project, the daily classroom work and the regular grade level meetings offered a natural context in

which the teachers could examine the above questions and concerns with their colleagues. Gradually over the first year the faculty had learned to use this context as a way to examine their assumptions about teaching math and to find the collegial support needed in attempting change. Teachers came to see this group activity as contributing to their own professional growth. As we moved into the second year of the project, the process of analyzing classroom work with their colleagues was a more familiar one and, through this exchange with their peers, the faculty were refining their work and considering math and pedagogical questions from their classroom in greater depth.

The broader discussion across the grades that had taken place during the previous year had encouraged us all to think about how children's understanding of specific concepts developed over the grades. Angela brought this up in the 4-5th grade level meeting as she discussed the work she was doing on area with her students. Talking with her colleagues previously, Angela had decided that it was valuable to consider how the area of a rectangle can be used as a starting point to determine the area of a triangle. She introduced this new approach to her class and found the work with her students on this concept very satisfying. As she described her students' efforts, she explained that, although her students had just a beginning understanding of area using triangles, this was a foundation on which they would base their learning about area with different shapes next year. Others too had similar thoughts. Alicia, a first grade teacher, had seen how much the younger children liked using cuisenaire rods during the demonstration lessons the previous summer. She and Cindy, who was assisting her, had begun using these rods in the fall to model the numbers 1, 2, ..., 10 and to look at different ways they could get a sum say of 8, using 2+6, or 3+5, et cetera. But Alicia had a question for her grade level group. "What math ideas are we working on [with these manipulatives], where are these ideas heading?" As Alicia thought about this question herself and shared her ideas with the group, they began to consider why learning sums to ten is useful in our number system and how the work with the cuisenaire rods could progress naturally to the model of the Base 10 blocks that was used in the second grade. Sally and Suzanne were working on similar questions as they used Base 10 blocks and beans and cups to examine the algorithms for addition and subtraction. By emphasizing the student dialogue around this work, they were making stronger connections for the students between the work with manipulatives and ways to record that work with symbols.

In this second year the grade level meetings continued as a forum for the faculty members to share their thinking about mathematical and pedagogical questions. In the grade level meetings we attended, this year, the teachers still sought our opinions. However they themselves often took the lead in examining the problems presented by their colleagues and discussing possible solutions. About half of the faculty were writing Teacher Reports and finding this a useful means of reflection about their teaching. These reports however did not provide the focus or continuity for the grade level meetings that I had hoped, and some groups had difficulty in finding a long-term direction of inquiry for their meetings. Despite this drawback, these discussions remained an important support for the teachers, especially for those new to the school.

Another shift in our roles was apparent as the teachers helped organize and lead the discussions in the Career Ladder Day sessions. These continued to focus on assessment. This year many of faculty found it more difficult to decide on appropriate ability levels for their students. Some suggested that their own ideas of math achievement were changing. During these sessions the faculty began to consider again the issue of school-wide ability grouping in math. As the teachers presented their concerns and understandings about ability grouping, we were impressed with their strength as a group to examine this sensitive question.

During the April meetings with Deborah Ball and Deborah Schifter we could see how the growth in our partnership and how the teachers' focus in their work in the project had changed as they debriefed after Deborah Ball's lessons. Deborah Schifter wrote in her report of that visit:

> "In the last round of debriefing sessions [held in April 1994 after a demonstration lesson given by Deborah Ball], I was struck by the attention teachers gave to the mathematics of the lesson. They listened carefully to what the children said and considered what these comments reflected about their understanding. Furthermore, the feeling that Deborah was a model who demonstrated strategies they should judge seemed to have disappeared. Rather Deborah's teaching gave them a context for reflecting on issues of learning and teaching mathematics. The fact that some teachers (John and Angela) chose to co-plan with Deborah, do the same lesson in their different classrooms, and then debrief together illustrates a dramatic shift in the teachers' relationship to Deborah, to each other, and to their work."

Reflecting on Our Work Together

The conversation we had begun rather hesitantly two years previously had grown. Our discussion had stumbled at times and gotten sidetracked now and then. We hadn't come up with a school mathematics program guaranteed to succeed, nor a definitive plan for educating preservice teachers. Questions remained and certainly others would arise in the future. But what we *had* done was to establish an effective dialogue, one supported by both math content and pedagogical knowledge, and one that we had come to see as a critical step in improving mathematics education at all levels.

At the third Career Ladder Day (beginning of April 1994) we all met in the library to talk about the work in the project. Carol Lubo led the discussion as the faculty reflected on their experiences during the last two years. The teachers were pleased with the changes they had seen in their students' learning. They noted that they too had benefited from their work in the project in a variety of ways. They explained that they were more comfortable with math as a subject and that some of their "math mysteries" had been solved. They took more risks "mathematically" now and they felt a greater understanding of the connections between the topics they taught. Rather than covering many topics lightly, they were better

able to examine the more important ones in depth. Because of their focus on student thinking and discourse, they felt a greater communication between themselves and their students and they had a better understanding of their students' development. The project work was not an easy or straightforward process. As many noted, considering different approaches and viewpoints did require time, commitment, and a willingness to take risks. But the faculty reported that their work in the project had brought them together as a community. They had developed a common focus in mathematics, and their professional growth in mathematics was extending to their work in other areas of the curriculum.

Listening to the comments I was struck by the variety of the teachers' images both in what they saw as effects of the project and in what they valued for themselves from their involvement. These images illustrated for me the richness and the complexity of a collaboration where the goal is to foster a real conversation on mathematics and the process of learning. I too had learned a great deal from our work together. The last two years at Washington Elementary had helped me to develop a better understanding of what it means to support mathematical growth across the grades. It helped me to appreciate the need for collaboration among all levels of teachers of mathematics. Thinking about the mathematical and pedagogical questions that elementary school teachers must consider had certainly impacted my own teaching goals. Moreover, my classroom experiences would continue to influence the direction of my mathematics courses for elementary teaching majors.

Sharing our experiences, we thought ahead to the future. Was it reasonable to view this project as only the first stage in an on-going exchange between school and university faculty? We had each gained from this collaboration. Could such a conversation continue? The intensive work of the last two years would have to change. What structures could be put in place that would support continued inquiry into the teaching of mathematics?

Herb had asked this last question at the final meeting in the library. In the ensuing discussion, the teachers stressed their need for regular opportunities to examine their work in mathematics with their colleagues. For this purpose, the school faculty voted to continue their grade level meetings in the coming year and to hold several half-day workshops on math. They set aside school money for release time and to help support a math specialist to organize the meetings and introduce new faculty to the mathematics work in the school. In the two years since the EMTTP project ended, the faculty has held grade level meetings. They did this regularly during the first year after the project. Because of a district-wide shortage of substitutes they were able to hold meetings only in the last trimester of the second year. This past year they have also taken part in a year-long project in reading, organized at their request along the lines of the EMTTP project.

The links established between school and university faculty have also been maintained. Preservice teachers in my university classes regularly take part in mini-practicuums in the classrooms of teachers who participated in the EMTTP project. In addition, several teachers have continued to be involved in other math projects and in graduate work in education (see Chapter 1). Those of us at the university have continued our efforts to

restructure the mathematics preparation for preservice students and to seek other avenues for exchanges between school and university faculty. Although our conversations about math are less frequent now, for those of us who became a part of this joint effort, the impact of the project work continues.

CHAPTER 6

PERSONAL JOURNEYS

The story of a partnership necessarily involves different pictures, varied images of the work that has taken place. In the first five chapters I have described how the partnership between school and university faculty through their work in mathematics during the EMTTP project. Although I have attempted to give a full and accurate account based on the actual events and illustrated by comments of the participants, what I have described is and must be my picture of a complex joint effort. This chapter will present the "Personal Journeys" of four other participants in the project. The first three stories are offered by three teachers, Diane Hunter, Carol Randell, and Scott Smith. Each teacher has given careful thought to their experience in the EMTTP project and to the particular aspects of their work that have been important to them and which they believe will help explain to the reader the nature of the project experience. The fourth story is written by Lorel Preston, graduate assistant on the project, who is currently completing her doctoral thesis in Mathematics Education. Lorel will describe how her work in the project led to her thesis topic and the implications she sees for collaborations such as the EMTTP project.

A New Understanding of Numbers

by

Diane Hunter

I was somewhat unclear about the purpose of the EMTTP project as the first summer workshop began in August of 1992. Having previously attended many workshops on a variety of subjects, I just assumed this one would follow the usual format, and I would come away knowing not only the newest but the best approach to teaching math. In a sense this was true. The approach many of our faculty used would change. However, this was not a "how to do it" class in which we learned step by step the methods we should be using to teach math. Instead it was the beginning of a process of discovery, one of learning not only how to approach math as a teacher, but also one in which we rediscovered what it is like to be the students, and how our students learn best.

We began by selecting a particular mathematical concept on which we wanted to focus. I chose "Number Systems" which was one of the classes Anne Roberts was teaching. As I took on the role of student, I soon became involved in a great deal of higher level thinking and reasoning. As we were immersed in trying to reach solutions, I came to the realization that there isn't always one way to get a correct answer as I had learned in the step by step processes I had been taught. We were finding several ways to get the same answer, and they were all right! Now a world of numbers was opening up to me as my own understanding of them grew. I would find this happening over and over again as I learned alongside my students.

As the school year started I was uncertain of how to begin my math program. I knew I wanted to focus first on addition and then subtraction. How could I help the students have the same kind of learning opportunities I had experienced during the summer workshop? Here our grade level meetings became a valuable resource for me. This was a time when we discussed our goals and the progress we were making in our own classrooms. As I listened to other teachers' ideas, I began forming some of my own and adapted what I thought would work best for me as well as my students.

After assessing the children, I knew they had an understanding of simple addition, but some were unclear about the concept of regrouping or carrying. I decided to use another teacher's idea, but in a different way. We drew pictures on a place value chart showing the ones as circles, the tens as triangles, and the hundreds as squares (Figure 1 below). It then seemed easier for the students to see that if there were ten ones (or ten circles) being added together, they could be grouped together in exchange for one triangle as shown by the arrow. Creating a new "ten" was shown by drawing another triangle in the appropriate column. I had the students write the addition algorithm by their picture. When a step was completed in the picture, it was also completed in the written algorithm with the hope of the student connecting the two and eventually reaching the point of not needing the picture to help them.

It worked! I was excited to see the students reach a greater understanding of the process involved.

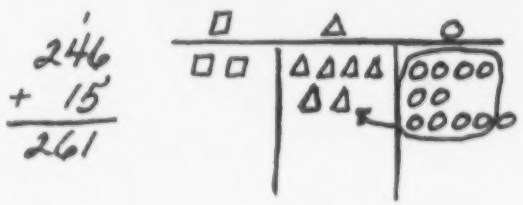


Figure 1

If the picture worked with addition, surely it would work with subtraction. However, I decided to begin teaching the concept of borrowing by using cubes. I wondered if this would be even more helpful in showing how a "ten" could be broken to make ten ones. It proved effective to a point. After a couple of days it became obvious that many could not get past the point of just playing with the manipulatives rather than using them as I had intended.

I returned to the idea of a picture, this time by crossing out the number of shapes being taken away. If there were not enough ones or circles to take away the needed number, a "ten" was broken by crossing out a triangle, then drawing an arrow to the ones column and drawing in ten more circles (Figure 2). Again, as was done in addition, the written

algorithm was placed next to the picture and worked simultaneously as the picture was drawn.

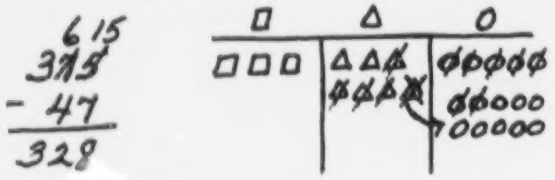


Figure 2

The results of this process were even greater than I had expected. I was truly amazed at the response of the students when I introduced subtracting with zeroes in the ten's or one's place of the top number. Because of the understanding in breaking needed tens or hundreds to create more of whatever was needed, the students automatically knew what to do. All I did was provide the problem, and they were able to immediately see the solution. "That's easy", they said. "Just skip over and break a hundred first." What had taken weeks in previous years didn't even need to be taught!

I was delighted at the level of understanding my students demonstrated. Also, I found myself gaining a greater understanding of processes which I had quickly computed to this point in my life, simply following the steps I had learned of crossing out, putting a one in front of the next digit, etc. It was all beginning to make more sense for me!

In teaching multiplication, I knew I wanted first for my students to have a good foundation in understanding the meaning of this operation. Anne Roberts suggested that I might find some of Marilyn Burn's ideas helpful in her book, *Math By All Means*. I especially found the Circles and Stars game a good way for my class to grasp the idea that multiplication is finding out how many there in groups or sets. The children worked with partners, taking turns throwing a die to determine how many circles to draw, then tossing it again to see how many stars to place in each circle. Each student made a booklet containing seven pages in which to play the game, so this process was repeated seven times. At the bottom of each drawing they were to write the total number of stars they had on a page. Then all the numbers were totalled. Counting the stars on each page easily worked into writing the multiplication facts. For example, if three circles had been drawn with five stars in each circle, the students wrote 3 x 5, meaning three groups of five. We were soon discovering that we also had fifteen stars total on a page showing five circles with three stars in each, or 5 x 3. The pictures were different, but the answers were the same (Figure 3).

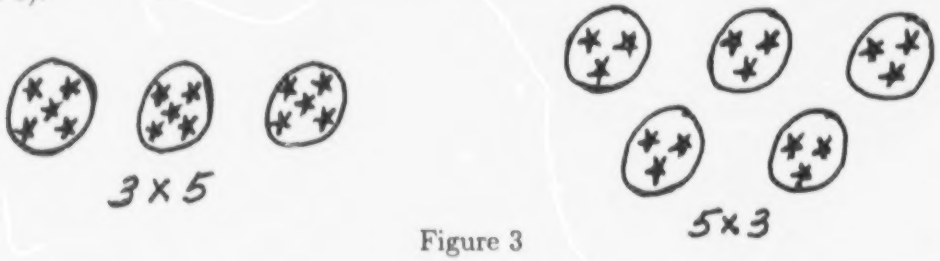


Figure 3

This also proved to be an effective way of visualizing that multiplication is repeated addition. However, even with all these added helps in knowing how to draw or figure out through addition the answer to a multiplication problem, the students soon realized that it would be much quicker to know the multiplication facts. Then some of the tedious work could be eliminated and the process quickened. The Circles and Stars game opened up, too, a way of naturally introducing division. Using the same pictures we could see that if there were 15 stars divided into 3 groups, 5 would be in each group. This even worked for remainders. When showing 27 stars divided into 5 circles, 2 were left over.

In a grade level meeting, another teacher shared her success of using graph paper as a way of visualizing multiplication and division. I decided to try it. The first number being multiplied could be traced across the correct number of squares, and the second one down. Then the rectangle was completed to show the total number of squares. Again, the students were able to see the difference in the pictures of 3 x 5 and 5 x 3 (Figure 4).

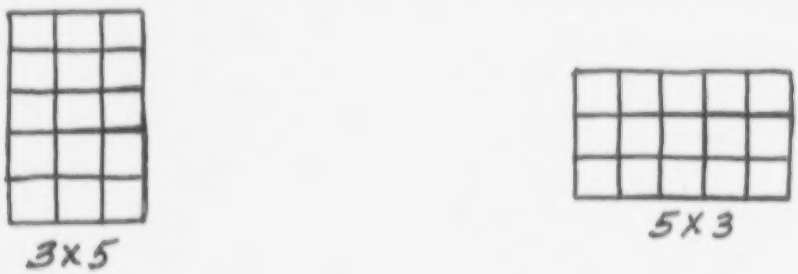


Figure 4

The graph paper also lent itself well to showing division (15 divided by 3, or 15 divided by 5 could be seen in the same picture). With this background we were ready to move into fractions. Fifteen divided by three could also show that 1/3 of 15 is 5.

Throughout this process, I again found my own level of understanding increasing. I had never really given thought to the difference between 3 x 5 and 5 x 3. As we worked with division, I even began asking questions myself. Why was it that the picture looked different when showing 15 pieces of candy being divided between 3 people and 15 pieces of candy being divided into groups of 3?I was confused until I was able to find the answer. There are two ways to look at division, and the picture will be different. In the first one we are looking at how many pieces of candy are in each group, and in the second, how many groups can be made with three pieces of candy in each one. This was something which had never occurred to me before, because I have always just computed the division problems following the steps I've been taught. Now I was finally beginning to understand this operation.

While I was meeting with success in many of the approaches I was taking, not everything turned out as I might have hoped. During our work on subtraction and understanding it as a process of "taking away," I decided to see if we could also subtract by counting on

a hundred's chart. Knowing which direction to move as they counted and where to start as well as stop became very confusing to some of the students.

After spending several days on this activity there was still some uncertainty, but I felt that the time had not been a total waste. The students had gained an increased sense of "taking away" and perhaps had even begun to understand that subtracting is also finding the difference between two numbers.

The EMTTP project had affected greatly both the teaching and the learning in my classroom, not only the learning of my students, but my understanding as well. I have learned to think of mathematical concepts on a higher level as I have participated and taught in classroom settings. It has been helpful for me to be reminded of the expectations we have for our students as they explore and discover. Because I have given greater thought to the reasoning of some things, for example, borrowing in subtraction, I feel I now understand some of these concepts much better myself.

I have left the "textbook approach" behind as I have become more confident in determining for myself my students' needs and understanding. Rather than teaching lessons in a book as they appear consecutively and then working through an entire page of problems for practice, the textbook has become just one of many resources from which I draw. I now use several books, ideas from other teachers, and perhaps most importantly, the students themselves. As I ask questions of "Why?," "How?," and "Could you explain your answer?," the students are gaining a greater understanding as they explore and ask questions too. The environment seems more secure as we discover the many ways to solve a mathematical problem, not just the one the book demonstrates!

Math has become fun as I have gained a greater understanding of how numbers and operations connect to bring about solutions. It continues to be my hope that my students will experience and see math in the same light, and that through our experiences of learning and discovering together, they will leave my classroom equipped with a greater foundation and understanding of the world of numbers and a desire to continue exploring.

Changing Math Practices Through Discourse

by

Carol Randell

My participation in the EMTTP project has started a gradual process of change in my math teaching practices. The most notable changes have occurred in the amount of discourse in which I engage while preparing, presenting, and processing my math lessons. During the planning stages I have discussed various approaches with university support personnel, then I have presented the lessons to my class with an emphasis on dialogue. Finally I have processed what occurred with my teaching peers and university colleagues. I have found this discourse invaluable because it has provided a window into the thinking of both my students and myself.

I received my first exposure to the use of discourse while observing Deborah Ball teach a demonstration math lesson at Washington Elementary during the summer EMTTP workshop of 1992. Her style of questioning required that the students explain their thinking and take ownership for finding a solution. She established an accepting environment where students were engaged and willing to risk. Mathematical tasks were presented and followed with open-ended questions that required students to clarify and justify their ideas. As I watched I wondered if I had sufficient mathematical knowledge to direct this style of discussion. I was intrigued, and knew that I wanted to learn more about this method.

During the following school year I taught a fourth grade math class grouped by ability. Herb Clemens was my university support person and he visited my classroom every few weeks. With his encouragement I tried to initiate discussions where students were required to explain their thinking. I also watched Herb conduct some discussions. I had mixed feelings about the results. I felt it was difficult to keep the conversation directed while managing the class. Several students had behavior problems and several others had low self-images regarding their math abilities. They were not used to this style of instruction and they did not seem particularly interested in sharing their strategies for solving problems. Some even asked if I would just tell them what to do. I persevered, but often felt frustrated. I found myself questioning my practices during this year but did not always feel prepared to make changes. As the year ended, I panicked about not having taught 2-digit multiplication. I pulled out all of the materials that would help provide a visual representation and then decided that I did not have time and would just show the algorithm. I even discussed this with the class, telling them that this time I was just going to show them the steps they needed to do. (Some sighed with relief.) I demonstrated the steps, provided several worksheets, and most of the students were able to do the problems with at least 75% accuracy. Unfortunately this did not hold true two weeks later when I gave a test. Many students did not understand the basic concepts and consequently could not rethink the procedure that they had memorized. This failure renewed my belief that it was a critical part of understanding to be able to explain the process when solving math problems. This year of thinking and evaluating my old teaching strategies was important in my process of change. I was not prepared to dive in immediately. I needed time to

weigh the effectiveness of the new things I was trying with the old methods I had used before.

I returned to the second EMTTP summer workshop with a clearer notion of what I wanted to accomplish. My goal was to learn how to engage the students in meaningful discourse about their mathematical thinking. This would in turn help me evaluate their level of understanding. I decided that a good way to achieve this would be to carefully observe the instructors who were teaching us during the first two weeks. I selected Anne Roberts' class in fractions primarily because I wanted to observe her questioning technique. During our interactions over the past year I had always felt comfortable asking questions and "risking" with her. I wanted to see if I could figure out what she was doing that allowed me to feel safe. As I watched, it became apparent that Anne's rapport was based on a whole style of interaction. Key elements were:

- Treating students with respect -starting on time -being organized -having a plan of what should be discussed but diverting when appropriate -maintaining an atmosphere in which students' ideas were valued
- Helping students focus and clarify their ideas -getting below the surface -checking for understanding
- Providing solid mathematical knowledge -making connections with math principles and beliefs

These elements seemed critical in establishing the kind of environment necessary for meaningful discourse to take place.

I continued this focus of watching how we were being instructed during the second week of the summer workshop. Here an experience of what not to do clarified what kind of questioning was productive. I was engaged with several other teachers trying to solve a math problem when we were interrupted by a visiting instructor. It felt like we had to stop our process of thinking in order to explain what we were doing. The whole intent of the questioning seemed to be to check on us, not to stimulate us. It felt intrusive and became irritating when the visitor took a pencil and started showing us what to do. We no longer felt a sense of discovery and ownership and quickly lost interest in finding a solution. (I think I reacted so strongly to this because it made me think of the times that I had been guilty of the same thing with my students. It was interesting to feel this from the student's perspective and I hoped not to repeat it.)

I started the next school year determined to change by incorporating more meaningful dialogue in my math classroom. My faculty stopped ability grouping for math so I taught my own second graders who came with a wide range of abilities. I immediately started engaging the students in math discussions during the daily calendar time. This was a 10-15 minute discussion each day that revolved around a calendar, counting tape to count the days of school, and other elements such as clocks, coin counters, and graphs. The children seemed to feel comfortable explaining patterns, exploring relationships, and communicating

their thinking. This carried over into explanations during the regular math lessons and my confidence level grew. My fear of losing control of the discussions subsided. This discourse allowed me to get more information and adjust my teaching appropriately but did not necessarily make me relinquish control of the lesson.

My university support person during this second year was Anne Roberts. I found the discourse that we engaged in very helpful throughout the year. She provided advice on mathematical questions, direction on possible next steps, and encouragement when I felt frustrated. She conducted some model demonstrations in my classroom and guided me in my thinking about the critical math components. This may seem surprising, but stopping to think about what math concepts I wanted to convey and what would be the best approach was a big step. This "thinking" about the math involved in every task was new for me. I did not feel so bound to go through the book in a lock step manner. I started feeling more critical in my selection of activities and what would best achieve my mathematical goals. An example of one of our conversations follows:

Anne was working with a small group of students on addition of 2-digit numbers. She started by having the students make an estimate of what would be a reasonable answer. She then had them do the actual calculation. Most of the students guided Anne through the process by telling her what to do with the tens first, then the ones. This concerned me because it had been drilled into my head to always add the ones first and then the tens. Later when Anne and I talked about the lesson I brought up my concern. Anne suggested that when people do math mentally they usually deal with the larger group first and this was the natural way for children to approach a problem. She predicted that when it became cumbersome or more time-consuming because of adjustments that needed to be made, the students would change their approach. I saw the sense in the first part about mental estimations, so decided to trust in the second part that the students would change when they saw a need. This was one of those lessons from my own background that I had never challenged. Having someone to talk it through with made it possible for me to reexamine my beliefs and change my thinking. It didn't take long for this to pay off. We soon had a new strategy that the class dubbed "Emma's Way".

One day when Emma was explaining how she got her answer to a two-digit addition problem she said she always started with the ones, just in case she had more than ten. This way she did not have to redo the tens column if she had to move some over. She felt like she had discovered a shortcut. The rest of the class seemed to think this was a good idea and we called it "Emma's Way" whenever we used it to work addition problems. The power of this exchange seemed immeasurable; this discourse in the classroom was starting to make sense. It was so much more effective than describing a procedure. The students saw a need, discussed a process that worked, and felt ownership. This sense of ownership was a critical element. The students viewed themselves as problem solvers who could make important discoveries. They were not looking for me to provide the "right" way.

The final area in which I experienced the value of discourse was in processing classroom lessons with my teaching peers. This step did not happen initially because I teach in a different school from most of the other participants. I did not attend a grade level meeting

with the Washington staff until late in the second year. When I finally did attend a meeting, I found it very helpful to talk to other teachers who were struggling with the same issues. I realized that I found this kind of exchange beneficial and I decided to pursue additional opportunities in this direction. This led me to participate in a math network with several other EMTTP participants locally and six school districts nationally. Having discussions with other teachers helped me reflect on my teaching practices and provided me with a support system while attempting to make changes. One conversation that illustrates this occurred last fall with Scott Smith, a teacher from the EMTTP project. I was concerned that I could not seem to get my math students discussing problems like I had the previous year. Scott expressed some of the same concerns with his group. As we talked both of us realized that our expectations of what should be happening with our current students were based on what we had experienced with our former students. The previous year we had been trying something new and were generally pleased with the outcome. Now these experiences were getting in our way. We had seen the value of discourse and perhaps we were comparing students from the end of the year with students just beginning the year. Possibly some students had not figured out what we valued in our respective classrooms or were still afraid to risk. We also realized that class compositions could make some years work better than others. Both of us had at least one negative student whose potential comments could pose a threat to other class members. We decided that we needed to keep working on the environment and the expectation to explain the thinking process one goes through in solving a problem. This conversation helped me clarify my thinking and made me realize how important discourse with peers can be.

The changes that I have been making in my math practices have been gradual. The discourse in which I have engaged with university personnel, students, and teaching peers has been critical to this process. It has helped me clarify my thinking and has provided me with a better understanding of my students' thinking. I plan to continue trying new ideas and learning new strategies for improving meaningful dialogue in my math classroom. I still have many questions about managing this kind of discussion. When to tell, when to redirect, and when to move on continues to be a struggle. I am also concerned with how to include the entire classroom in the process. As I try these new methods I seem to create new questions. I think this journey has just begun.

The EMTTP Experience

by

Scott Smith

The principal approached me in the hallway during the spring of 1992 and asked if I would be interested in attending the month-long EMTTP workshop during the upcoming summer. I tried to hide my sigh of reluctance, and feigned some token interest by asking a few questions about it. Another commitment - another chunk of precious summertime spent sitting in class did not sound appealing. I had attended a multitude of workshops during my 20-plus years of teaching, usually gaining a good idea or two and maybe some eye-catching reproducible worksheets to use, but the means didn't always seem to justify the ends when one compared the hours invested sitting in a workshop with the limited number of useful ideas that I might utilize.

"Almost everyone else has signed up," said the principal, who knew that I probably wouldn't be too comfortable being a lone dissenter, "and there is a very decent stipend offered." Well, that was a pleasant surprise.

Without enthusiasm, I told the principal she could put my name on the list.

By mid-July I really regretted my acquiescence. I had just taught six weeks of remedial summer school and would have only one week free before beginning the EMTTP project. Why had I said I would participate? I was a good teacher - a good math teacher. Although I hadn't taken much math as an undergraduate or while working on my master's, I had later decided that a mathematics endorsement on my teaching certificate would make me more marketable, so I returned to campus and took 32 credits of college math. I didn't need or want any more math education, and didn't want to spend the last month of summer spending hours impersonating the role of children playing math games, or listening to a workshop facilitator extolling the virtues of their latest can't-teach-without-it booklet of "101 MORE Ways to use Egg Cartons in the Classroom." However, I had made the commitment. And it would be fun to spend time with others on the staff, so I decided to make the best of it. Still, my attitude remained poor as the workshop began.

The first couple of weeks were okay. Herb and Anne are both genuinely nice people - they didn't project themselves as the all-knowing professors. They gave us several choices of content areas to study, and they both used very low-key teaching styles. They presented problems to investigate, then let us pair off or team up to work. It seemed a little odd (and time consuming) that they didn't spell out how we should do the problems. They just threw a quandary out there, then after giving us time to work on it, wanted to hear the details of everybody's thinking. It certainly was not an efficient use of time - I mean maybe we would only talk about one problem the whole session. I figured they must not have planned enough, so they really stretched things to fill the time.

Oh well, we were getting paid - and the mathematical situations they presented were quite interesting. I didn't want to admit it to anyone at the time, but when we worked on

estimating a value to use for Pi, I had never actually known before what Pi represented. If any of my former teachers had ever given an explanation of Pi's relationship to a circle and its circumscribed square, I must have been absent that day. Drawing it out as we did on graph paper, it became so logical and apparent. I experienced one of those "Duh!" hit-yourself-on-the-forehead moments. Why hadn't I figured it out before?

So the first two weeks were more tolerable than I anticipated. Gaining some new mathematical insight had been fun. Even though we hadn't covered a lot of material, I enjoyed the way Herb and Anne and others involved in the project treated us. Rather than "teach" us, they gently allowed us to figure things out on our own, which was much less demeaning to us than it otherwise might have been. (Was this the all-knowing PhD professors allowing the lowly grade school teachers to maintain some sense of dignity? If so, it worked.) Anne and Herb were also endlessly interested in hearing how we might present particular material and mathematical ideas to our students. They said that we were the experts in teaching children and they hoped to learn from us. I figured this was a roundabout way of saying that we were all guinea pigs in some research project they were doing, but nevertheless, being called experts and professionals felt good.

With the latter two weeks of the project approaching my enthusiasm again waned. There was so much to do to get the classroom ready. And who were these two Deborah people that were coming anyway? Maybe they would be the egg carton pushers. We were supposed to watch one of them teach? Big deal.

Surprisingly, Deborah Ball used a similar approach with volunteer students that Herb and Anne had used on us. She presented a mathematical situation, then wanted to listen to the kids explain their reasoning. Naturally, some of the reasoning was erroneous, but she seemed to have a real knack for turning incorrect answers into learning situations. She seldom told the kids they were wrong, but kept delicately prodding them to expand their thinking until they often saw the errors of their ways on their own. Again, this approach allowed them to maintain their dignity, and permitted them to feel like they were actually good at math. Well, maybe they could be if they somehow could keep believing that, I thought. That would be great. But this was all an artificial environment. This wasn't real school - with real curriculum. These students were attending voluntarily. There was no threat of grades hanging over their heads. This class was a short diversion during the long dog days at summer' end - unlike a regular school day when students (and teachers) are sometimes overwhelmed with the responsibilities of all the other subject areas too. And no one was dictating to Deborah Ball that she must cover X amount of material during Y amount of time, or that the students would need to perform adequately on a standardized test as a result of her teaching. Oh, I had all kinds of rationale why this model teaching might be equated to a Disney production.

Nevertheless, the summer session had turned out better than I anticipated. It had been a worthwhile experience, but now the school year was starting and I had to get back to the business of teaching.

I was told we were going to continue meeting with Herb and Anne periodically throughout the year. Darn - they would probably be expecting us to incorporate some of the stuff from the workshop into our classroom teaching. Yet they hadn't even given us reproducible worksheets or a list of games that kids just love to play.

It was at that point that I went back and read some of the initial letters and paperwork that described the project. It was to last two years! What had I gotten myself into? I received the first stipend check and was so very tempted to just take the money and run. All the facetious thoughts I'd had about the project suddenly became genuinely negative. I mean, the four weeks had gone better than I anticipated, but when a workshop is over, it is time to glean out the few useful remnants and label and shelve the rest. I didn't want to burst anyone's bubble, but there was no way that a teacher could get through the mandated amount of curriculum I had to cover if I went as slow as Herb, Anne and Deborah.

Our involvement during the school year turned out to be mostly a matter of having bi-weekly meetings with Anne and my co-workers who taught the same students and grade levels that I taught. At first it was very frustrating to plan for a substitute and walk out of the classroom in the middle of the day. Judging when to maintain routine and when to break it is one of the fine arts of having an elementary classroom run smoothly. Although I appreciated not having to meet after school, I resented this bi-weekly disruption in my schedule. The kids didn't like me leaving, so I often returned to a chaotic class and a substitute who insisted that Johnny be punished for being so rude to her. Gradually though, the bi-weekly meetings became part of the students' routine, and I requested a different substitute who didn't rely on a "just wait until your dad gets home" mentality.

I began to look forward to the meetings. I enjoyed the company of my colleagues, so it was nice to have extra time to spend with them. Thirty minute lunch breaks which consist of gobbling one's food so enough time is left to get to the bathroom, stand in line at the copy machine, check the office mailbox, and maybe try to call that insurance guy again and hope you're not on hold too long - well, not much time is left to socialize - much less discuss mathematical pedagogy. But the meetings gave us time. Time to compare notes and share ideas - time to plan how we might work together to improve the overall math experience for our students at Washington.

It felt awkward at first. We were so used to working alone. To question and share what goes on behind closed doors did not come naturally. It was a little like disclosing our personal finances or fears. But just like a bond that results from sharing intimacies, over the months we began to develop a deeper trust and respect for one another. We began to see each other in a new light, viewing one another as resources of information; as friends with common interests. Our math teaching became an interesting topic of conversation to us - after school, sometimes by telephone at night, or even during our frenzied and coveted lunch times.

With encouragement from Anne and others, we began to open our doors to each other. The project allowed money for substitutes to give us opportunity to visit other classrooms.

Again our cohesion was strengthened as we allowed ourselves to share our vulnerabilities. Our attention to improving the teaching of mathematics became more cogent as it became a shared pursuit - a common avocation.

As a result of the bi-weekly meetings and ongoing workshops during the career ladder professional days throughout the year, I gradually began giving more thought to my mathematical pedagogy. Somewhere along the line, during the early spring of 1993, I was converted.

I began examining my teaching of math. I thought back to my first years as a teacher and recalled the frustration and surprise I experienced concerning the amount of material that students did not retain. Scores on chapter tests were decent, but the end-of-year tests and the annual SAT math scores were quite poor. I remembered being perplexed as to why the students didn't seem to retain the mathematical procedures very well. After a few years in education I had rationalized away my concern and decided that my expectations were too high - that a certain amount of forgetting and regression was the norm. After all, why else did math textbooks generally present the same material over and over again, year after year? I became convinced that the key to mathematical fluency was repetition, repetition, repetition - repeatedly. Through the years I ignored those students who complained that they had "done this before." I took it in stride that those who were frustrated with fractions usually remained frustrated. Some would never understand ratios, I rationalized. And I decided that decimals would always be decimating to others. It is not a perfect world.

During that spring of '93 I also began to think about my own educational experiences in mathematics and the way I utilized mathematics in my personal life - and I realized that a great big gap existed. As a student, like all good students, I had taken notes in class; done the required exercises; crammed for tests to memorize formulas; and came out okay grade-wise. But I remember very few of those formulas now. So how do I manipulate numbers today? Well, an example that came to mind during my conversion is a humbling one. Like one of those words that a person always has to write down to make sure they are spelling it correctly, I never remember how to determine the interest rate on my savings. Given the amount of principal and the amount of interest dividend that has accumulated, I don't know whether to divide the principal by the dividend, or the dividend by the principal. Yet I can quickly perform both operations and then know, with certainty, which is sensible and correct. The algorithm for determining interest rates continually eludes me, but the conceptual understanding of how to manipulate the numbers to calculate the rate remains.

I thought about many other mathematical operations that I perform in my daily life and realized that the ones I am sure that I am performing correctly are the ones to which I can attach a conceptual understanding. Even though I was taught formulas for distance, rate and time problems, I seldom trust my memory of D = RT when estimating travel time, but invariably rely on my conceptual understanding to reconstruct meaning and assurance of reliability. After drawing the circle in the square to estimate π during the first EMTTP summer workshop, the mental picture of $A = (\pi)r^2$ made sense to me as a conception, not

just a formula. I could forget the formula completely and still be able to reconstruct it from my understanding - confidently.

I decided that if I could instill this same sense of confidence (not memory) in my students, I would be doing them the much larger favor in the long run. When faced with a mathematical situation in the future, they would not have to give up in frustration because they had forgotten the algorithm, but could confidently recall that they had figured it out before and could certainly do so again.

I decided that I really wanted to change my approach to teaching mathematics. I wanted my students to understand concepts. I wanted to increase their level of confidence in their ability to figure things out and to understand. I wanted them to reason for themselves and be able to explain their reasoning. I started thinking of ways I could incorporate similar methods of discovery learning in other subject areas. I wanted to quit preparing them for chapter tests and start preparing them for life.

Finally, after all these months, I wanted to be more than just a presence in the EMTTP project - I wanted to be a participant. I looked forward to the second summer workshop, and was thankful that I was going to have another opportunity to be part of it. For me, it had taken the first year to be convinced of the project's validity. It was during the second year that I seriously connected with what was going on, focused on what I could learn, and really began to incorporate the things I was learning into my classroom.

I began by working to establish an atmosphere of respect where students would feel comfortable expressing their thinking, whether it was correct or erroneous. I adopted a modified form of the Socratic method in my interaction with the students. I encouraged them to not look to me to validate or negate their answers, but rather to look at mathematical principles - and to look to themselves.

Initially I remained skeptical that the students from the lower-middle class neighborhood surrounding our Chapter I school would adapt to a more constructive approach and take more responsibility for their own learning. I was no Deborah Ball. But my skepticism gradually faded as most of the students responded positively to the "new" approach. They seemed to really appreciate "discovering" mathematics, and comments from parents about how much their child was enjoying math class and how much they seemed to be learning were encouraging to me.

I was extremely pleased (and surprised) with the way most of the students emulated the behavior I modeled, usually remembering to converse respectfully with one another. As a class we spent considerable time talking about how we could establish a non-threatening environment in the classroom. The students agreed to refrain from saying things like "that's stupid" or emitting moans and groans of disapproval and exhibiting other put-downs. I worked to replace "right" and "wrong" with "agree" and "disagree." Before long, some of the most shy students and some who struggled with mathematical concepts, became willing, even eager, to verbalize their thinking before the whole class because they

felt safe in the environment. I was proud of the way the students adopted these edicts of conversing. I sensed that they felt a newfound sense of pride within themselves toward their ability to be studious, successful students.

My students were not alone in their newfound sense of pride, for I experienced a new respect of my role as an educator. I desired not only to become a better math teacher, but to try to create ways to allow learning to be more meaningful in other curricula as well. I began experimenting with less autocratic methods of teaching in the other subject areas. Rather than handing out my list of "Commonly Used Prepositions" that I had depended on for years, I approached the topic by having the students examine the syntax in their own writing and discovering a purpose for "these little words that start those phrases." In social studies the students researched their own family trees, interviewed grandparents, and compared the customs and available technology to their own generation. This comparison served as a personalized and purposeful springboard to continue looking back in time and study history. I stopped giving students lists of states and capitals to memorize, and started showing them how to use resource materials and create maps to locate where their ancestors originated or where significant historical events occurred.

I began viewing myself as an apprentice in examining educational pedagogy. I felt I had a new perception of the art of teaching - and the act of learning. Questions about effective instruction and constructive cognizance which arose during this two-year EMTTP experience continue to be a driving force in my pursuit to enhance the educational experiences of my students - as well as my own.

Thinking about Teaching, Learning, and Change

by

Lorel Preston

I met Anne in the Math Department through our work in the preservice elementary mathematics sequence. We often got together, compared teaching notes, and talked about directions we wanted to pursue in teaching the math sequence. At the time I was also working towards a doctorate degree in mathematics education. So when Anne approached me about being her graduate assistant for the EMTTP project, although the role was not clearly defined, it sounded like an excellent learning opportunity and a new way to become involved in the current reform efforts to improve mathematics education. Because of the time involved in the project, however, my decision to assist Anne meant I would give up–at least temporarily–my role of teaching prospective teachers.

Several years later, now that the project is officially over, I am still working on that same doctorate degree and have resumed my work teaching preservice elementary teachers. On the surface it appears that little has changed regarding my own academic and professional pursuits; but that is not the case. As it turns out, by inviting the teachers to open doors to some ways of thinking about mathematics, they, in turn, opened doors and invited us into their mathematics classroom experiences–and a conversation aimed at developing a strong mathematics program began. This conversation, although slow to emerge, has been ongoing and constantly evolving. Several layers and overlapping strands developed over the two years. As a voice in this dynamic dialogue, the ensuing conversation impacted both my teaching and doctoral work in important ways.

When the project began, however, I had many questions about this collaborative effort to accomplish change and my role in it. One of the overarching goals of the project–improvement of the elementary mathematics education at the school–centered on creating a partnership with the teachers. But what really was a partnership? What should change look like? How would we know when (or if) it happened? Would it continue after the project was over? What role would I play in accomplishing change? As I look back, I now realize that knowing precise answers to these questions ahead of time was probably not realistic. Besides, because the conversation was constantly evolving, responses to these questions would have looked quite different at varied points in time. In my view the questions also would have yielded multiple responses dependent not only on the point in time, but on the strand of the conversation on which one focused.

One thing I did know going into the project, at least in theory, was that accomplishing change would take time. Learning to create the kinds of changes in thinking about mathematics education envisioned by the reforms is hard. Changes do not happen overnight. Nevertheless, in my initial excitement to work with the teachers I enthusiastically looked for change, at least changes in teacher's thinking, as a result of each meeting. I realized the frustration of this expectation shortly after the first set of biweekly grade level meetings. As a result of the summer session feedback and conversations with Deborah Shifter, we

decided that the initial focus of the meetings would be helping the teachers to develop a "big picture" of mathematics for themselves that several had sensed was missing. Anne and I saw the value of having such a map of the elementary mathematics curriculum and drawing upon it for planning and teaching. We envisioned that dialogue centered around this broad question of developing a big picture of mathematics might provide a fruitful context in which we could consider issues of math content and pedagogy. So this focus seemed like a reasonable place to start. The task, however, proved to be far more challenging than we anticipated. It was the start of many first-hand experiences that change, if it occurs at all, is often slow.

We still believed that having such a picture of mathematics mattered in teaching mathematics for understanding. Consider, for example, the responsibility of selecting and pursuing a worthwhile mathematical task. Being able to "see" the mathematics possible in its scope depends largely on the teachers own understandings of how a concept grows and relates to other parts of their mathematical map. Similarly, orchestrating such a task in ways that take advantage of students' unforeseen questions or conjectures also requires a good sense of the mathematical terrain. But from our perspective, it appeared only a few teachers came to the meetings prepared to talk about their view of the math curriculum at their grade level. Frequently the conversation got off the subject as teachers got involved talking about recent classroom concerns or anticipating upcoming ones. It was not uncommon for Anne and I to go into these initial meetings with a plan and leave without accomplishing any of the items on the agenda. It became clear that developing a big picture of the curriculum with the teachers was not an effective means for advancing the conversation. Looking back, however, although this task did not develop the conversation in ways that we anticipated, it provided an underlying theme for the project, one which we revisited explicitly several times during the two years.

Still, the question remained: What should we do to move on? We recognized more fully the need to actively involve the teachers in setting a direction for the meetings if the conversation was to be fruitful. Drawing upon their interest (and need) to discuss their immediate classroom concerns, we decided to incorporate math content issues within the context of the teachers' daily work and surrounding issues. To help prepare for meetings and provide some continuity from one meeting to the next, Anne sent a memo prior to each meeting that briefly summarized the previous meeting and provided a few questions to consider for the upcoming meeting. Often the questions to think about arose from issues we, as a group, decided to follow up on. Because we were beginning to visit teachers' classrooms on a regular basis, we were also able to build a conversation around specific events that occurred. With this smaller focus–on content and concerns with which teachers were currently dealing–we hoped to bring up important mathematical issues and eventually relate them to a bigger picture of their mathematics teaching. In setting this new direction for our meetings I began to see more clearly an important aspect of building a partnership–participants needed to share a focus and a need for even having the conversation in the first place.

Although this way of approaching the agenda allowed for a more focused conversation

on issues of mathematics education, I still faced challenges in defining my role in the partnership and understanding what creating a partnership was all about. For instance, during one of the grade level meetings some of the teachers were interested in talking about how to help their students go from knowing basic multiplication facts to dealing with something like 14 x 3 and then 14 x 23. Our conversation led to a discussion of the concept itself and what it "looked like." Several teachers became intrigued with exploring pictures of multiplication. In addition, we looked at several alternative multiplication algorithms that illustrated the partial products found in the pictures. The teachers became excited about these observations, but some were surprised that we suggested such explorations might be beneficial for helping students to develop their own mental picture of multiplication. Responses such as these initially frustrated me even though I knew it was not uncommon for a teacher (myself included) to know something about mathematics, and even be excited about it, yet for some reason choose not to incorporate that knowledge into his or her classroom practice. Part of my frustration was undoubtedly due to my eagerness to help teachers and see change happen soon. Even though I used to teach math in the public schools, it was only after spending time in these teachers' classrooms and really hearing what teachers said (and did not say) both in and out of the classroom that I realized a new appreciation of the complex issues elementary school teachers face–issues that include but go far beyond mathematics content.

One of these complex issues that teachers faced was assessment. Assessment interests and concerns came up almost from the beginning of the project and never seemed to go away. The interests were many and varied. For example, because most of the teachers grouped students in mathematics by ability, teachers were concerned with how to test students in order to get them placed correctly and quickly into three or four different ability groups. Later in the year sixth grade teachers became concerned with upcoming junior high school placement tests in mathematics. They wanted their students to do well but were not happy with the way the students had been tested and placed in the past. Sometime during the second year of the project teachers tried out a new state mathematics performance assessment that became available for grades one through six. Some of the third grade teachers experimented with creating their own placement tool–a blend of computational and conceptual items. One of the first Career Ladder Days was devoted to learning about and investigating alternative mathematics assessments. It turned out that the next one also focused on assessment. In addition to the formal assessments imposed on the teachers from outside their classroom, teachers dealt with concerns within the classroom, too. It certainly seemed to be a recurring and important issue.

I must admit that at first this was not one of the most exciting or important topics for me to consider–but it grew on me. I became quite interested when a discussion of placement alternatives from the beginning of the year led to a discussion of how one knows math and what math is worth knowing. This eventually led to ongoing discussions of different perspectives on what mathematics to assess and how to assess students on it. Teachers also questioned their purposes and reasons for assessment: What is assessment? What should it mean? What information do we want to know? What can I learn from an assessment? Who is it for? How can we educate parents about assessment? The third

grade teachers who had created their own placement tool came to the conclusion that one can never get a single assessment tool that is complete or that one may even feel good about. In reflecting on their experiences in developing their tool one teacher commented, "I have a new sympathy for test writers." I grew to admire the depth of thought that the teachers gave these assessment issues and gradually came to realize how profoundly assessment impacts their lives.

Looking back now I can see that my role in the project was evolving, as was my view of the partnership itself. Regular meetings with Anne, in which we shared our experiences and planned for upcoming meetings with the teachers, helped me to understand that although we might exchange and develop ideas with the teachers about mathematics and pedagogy, the teachers were not obligated to use our suggestions. On the contrary, we strongly believed that in order for new ideas to impact classroom practice in meaningful ways the teachers needed to find value in them for themselves. As Deborah Ball might say, we did not want a "superficial wearing of someone elses costume." Discussing these issues with Anne helped me to see that one aspect of my role in the project was to enable change, but I could not make it happen. The teachers shared in this responsibility. I realized that a partnership of this kind does not just happen by mutual decision. It is built a little at a time by individuals, each bringing his or her own sets of experiences, beliefs, and expertise. Consequently, the conversation affected each teacher in different ways and each made different contributions to the conversation. Thinking about change in this way helped my frustration to give way to a deeper comprehension of the concept of a partnership and of my role in the emerging conversation.

As the project progressed I became more of a "collaborator" or a "partner" in the project–not simply a teacher or a learner, although I did teach and learn. The lines between teaching and learning, however, were often blurred. One of my visits to a third grade classroom during the second year of the project illustrates this for me. Eric was exploring addition of fractions with his class. Eric had given them the task: How much is 1/3 of a pie plus 1/4 of a pie? Eric supplied paper and compasses and was willing to let things get a little chaotic as students excitedly looked for a solution. After a while several students had ideas to share at the board. Roberto had decided on 2/7 since there were 2 out of a total of 7 pieces. Charity took the two pies, "put them together" to make one big pie and took 1/7 of it–she justified this since it was 1 "big" piece. Anton took the 1/3 and the 1/4 and put the two pieces together inside another circle and found out that it was more than 1/2 but less than 1. At this point nobody had "the" answer. Eric, however, patiently posed questions in an effort to pull the mathematics out of the students solutions and conjectures. For example, after having the students make a pie that was cut into sevenths, the class decided that Roberto's solution of 2/7 was smaller than it should be to make sense. Emils demonstration supported this since 2/7 was smaller than 1/2, and the class had agreed from Emils demonstration that the answer must be more than 1/2. Eric had used each students work and came to the conclusion that each had a "piece" of the solution. Eric, however, did not show them the "right" answer; he left the problem unresolved and planned to continue it the next day.

Through a conversation with Eric after the lesson I learned that his students did not formally know how to add fractions with unlike denominators. They did, however, have a basic understanding of fractions. Eric was experimenting to see what his students could do with a problem that was just a little beyond their reach. He did not necessarily plan for them to solve the problem (at least not today), but hoped to bring out some typical misconceptions (as Roberto's solution illustrated) or underlying issues that students would need to consider in order to understand how to solve the problem. I was impressed by his willingness to try such an open-ended task with his students. After hearing about his goals and realizing where he wanted to go with the problem, I was able to suggest a new model for him to consider that might help with the eventual idea of finding common denominators. Eric appeared very interested in pursuing this approach and I ended up helping in his classroom as he pursued exploration of the new model with his students. I found learning about his thinking during our conversation and exchanging ideas worthwhile and became intrigued with both classroom visits and the opportunities to exchange thoughts with teachers about their experiences with new approaches to their math classes.

At one point in her reflection Anne asked, "Is this conversation beneficial, and if so, to whom and in what ways?" My response is "yes." The insights I gained through different strands of the conversation–with Anne, with the teachers, with myself–helped me to reflect upon my own teaching and academic pursuits in many ways. For example, the project significantly influenced my doctoral work. When the project began I had not decided on a dissertation project to pursue. After spending many meetings and Career Ladder Days listening and learning, though, I came to respect the teachers' needs to deal with assessment issues. I also became intrigued with the many questions surrounding the nature of mathematics that came up as a result of thinking about assessment. Questions arose such as: What should students know about mathematics? and How can we really find out about what they know? Many teachers in the project thought long and hard about questions such as these.I was impressed by their thoughtfulness and the fact that in some way assessment issues impacted them every day. When the state decided to implement a new state core elementary mathematics test, it seemed like an opportune time to integrate my growing interests in assessment issues with what was going on in the state. When the project began, however, I do not think I would have anticipated that my overarching research question would be to learn about the meanings teachers make of this new test.

The conversation also helped me to develop new approaches in my courses in mathematics for preservice elementary teachers. I grew to understand that developing a big picture of mathematics for oneself–particularly a picture in the spirit of the reforms–requires time to reconsider ones assumptions about mathematics. As a result of these experiences, I now encourage my preservice teachers from the beginning of the math sequence to think about and map out some of the big ideas they see in each of the mathematical concepts we explore. I also ask them to consider ways in which these mathematical concepts are related to each other, to other maths, to other content areas, and to real life. In addition, they consider how these concepts connect and grow across the curriculum. As the sequence progresses, students realize that their own picture of a concept and its relationship with other concepts may change, grow, and connect to other concepts in surprising ways. These

"concept inventories," as my students now call them, are considered works in progress. There is no single "correct" way for a student to construct his or her map and no end in its intricacy. Using this task as an underlying theme for my courses I hope that by the end of the three-course sequence students will take away a useful developing picture of mathematics–one that will enable them to meet the new challenges and directions of teaching children mathematics.

In addition to encouraging my students to think about the big ideas underlying and related to the mathematics they will teach, the conversation reinforced a need to integrate into the preservice mathematics courses both content and pedagogical knowledge within the culture of an elementary classroom. That is, I realized more clearly that having a strong picture of elementary mathematics from which to draw may be necessary for selecting and pursuing worthwhile mathematical tasks, but successfully orchestrating such a discussion or exploration around the tasks involves much more than subject matter knowledge. Knowledge of children and their mathematics is essential. I have found, however, that integrating these ideas and helping prospective teachers to relate to their complexity is not an easy task. I continue to struggle with this. My experiences in the project, however, have provided me with some ways to get at these ideas with students. For example, samples of student thinking and student-teacher interactions serve as cases for my own students to consider. Several teachers have also allowed my students to visit and work in their classrooms. But perhaps one of the most beneficial contributions from the project to me has been the teachers' willingness to continue different strands of the conversation even though the project has officially ended.

CHAPTER 7

LOOKING BEYOND THE EMTTP PROJECT

In this chapter Herb Clemens, the EMTTP project director, and Carol Lubomudrov, the principal of Washington Elementary, consider the work of the EMTTP project. Herb examines the position of such projects within the school/university community and he discusses the difficulties of maintaining support for similar projects in mathematics education. Carol looks at the question of professional development for school faculty and describes the impact of the EMTTP project on the learning environment of the school community.

An Amateur Amid Professionals: A mathematician on the loose in a schoolhouse

by

Herb Clemens

It was with the noblest of intentions (and maybe just a bit of naiveté and arrogance) that I entered into EMTTP. Like a teenage lover believing that it will all work out if we are just totally honest with each other, I decided to replace my uncertainties about continuing to do research mathematics with certainties about teaching mathematics to kids. After all, I had worked with kids and made all these great discoveries about experiencing math again together with those discovering it for the first time! I had worked with teachers, whose love of their work and selfless dedication inspired me. I was ready to try work in a schoolwide project!

It's now four years later. I have a strong feeling of accomplishment, not so much about my own work in the project, but of the success, importance, and, at some basic level, the correctness of EMTTP. On the other hand, I have the strong feeling that the two institutions external to Washington School which control the destinies of our partnership, namely the University of Utah and the Salt Lake City School District, emerged unscathed from our encounter at Washington School. In fact, before too long the winds of change will cover the last track left in the sand at either institution. (It's reassuring to know that no lasting damage was done!)

I'm not saying all this as a lament or an exercise in cynicism. Those three or four of us university teachers who opened ourselves to the alien world of the elementary classroom profited enormously, roughly in proportion to our willingness to make ourselves vulnerable, expose our ignorance and ineptitude, ask, listen, and work hard. Those 10 or 15 of the Washington teachers who opened themselves to the somewhat alien world of more "advanced" or "conceptual" mathematics profited enormously, roughly in proportion to their willingness to make themselves vulnerable, expose their ignorance and ineptitude, ask, listen, and work hard. In that respect our project, with very few answers or hidden

agendas, was slow enough, careful, patient, and caring enough to succeed. The "tracks" left in the minds and hearts of many of us participants are not tracks in the sand but the makings of more permanent patterns of thought and emotion.

However, as an exercise in institutional or, to use one of the recently fashionable words, "systemic" change, we must in retrospect call EMTTP a failure. A rich consensus emerged from our efforts all right, that our partnership was valuable to most of us and that it was a solid and potentially lasting basis for real improvement. But our partnership was also necessarily costly in terms of human and material resources, and it was necessarily slow, patient, and individualized. This latter reality was of course utterly, if unconsciously, rejected by the "system" on both sides of the aisle, school district and university. Appropriate lip-service was paid, of course, by all concerned. Intentions were good. But "systemic" realities necessarily set in. "Systemic" realities replaced the principal of Washington School before the process of reform itself became "systemic," before the habits of school-wide discourse and collegial reflection on curriculum and practice themselves became the norm. "Systemic" realities drove some of the university participants (yours truly, for example) back to more traditional academic pursuits in order to defend salary and status. One is reminded of the Tom Lehrer line on the Spanish Civil War–"They won all the battles, but we had all the good songs!"

Again, it is not a question of "shoulds" or "blame." Institutions do as institutions are. By their nature institutions defend the system which established them and which rewards obedience to its norms. Individuals change, societies change, and eventually those changes force institutions to incorporate them or to fail. But in education, both in our school districts and in our universities, we are still a long way from such an apocalyptic moment. And the levers of educational change which do exist, levers completely out of reach for us participants in EMTTP, in fact lie outside school districts and universities.

All of the above not withstanding, I found something quite noteworthy in our elementary mathematics education reform work with teachers, something which to me makes success at the "grass roots" and "systemic change" almost antithetical. The efforts which I often found to be most productive, which my instincts and intellect responded to as most effective and rewarding, were those furthest removed from the levers of policy and system. To be respected or even noticed by those who make decisions and those who provide money and institutional status, reform efforts must be quantitatively measurable in the short run, they must have a well articulated and persuasive philosophy backed by empirical research, and, all too often it seems, they must promise far more than any reformers, to my knowledge, have been able to deliver. On the ground, working with teachers in a single school, however, the indefinable, quantitatively unmeasurable change of mathematical ways of thinking, the change of self-concept as a professional, the change in ability to listen to and discourse with students, comes about one teacher at a time. It never comes twice in quite the same way, never twice according to the same timetable, never twice in the same conceptual or philosophical framework. The same virtues of listening to students, really listening, and of appreciating very diverse ways of thinking, understanding and expressing,

apply even more strongly in the intellectual and emotional interaction when school and university partners work together.

If we had 30 teachers working in EMTTP, we had 30 programs, because we were dealing with 30 individuals whose common mathematical base was far less than met the eye, whose ways of thought, of reasoning, of understanding the meaning of their teaching, had so very little in common. If there were four or five of us university folks, no two of us shared the same understandings of mathematics, or of education either. Some of us were constructivists, others symbolic reasoners, some with traditional tastes and habits, others disdaining tradition, some disciplined of thought and speech, others less so. In short, neither party at the table, neither the university people or the teachers, is equipped or really disposed to employ the kind of sophisticated, highly nuanced tools offered by educators and education publishers. Both groups, however much they disguise it, have rather elaborate agendas based on their own very particular and subjective ways of thought, of reasoning, of understanding the meaning of mathematics and its learning. As we proceeded in EMTTP, the thought of some single philosophy of learning, some "right way" to teach or think or share quantitative and spatial experience, seemed less and less realistic or useful.

There is, of course, the need for shared beliefs and purpose in any collective human enterprise–religions without a body of beliefs, even without angels and devils, do not seem to do very well in the long run. So, in mathematics education, we need our Standards and our Constructivist Philosophy. But from my perspective these don't get one very far. "Adopting the Standards" is perhaps useful as an educational ritual, but codified, uniform Standards, however enlightened, are necessarily inadequate in the face of the very personal, intimate, and diverse ways human beings come to perceive quantity and space.

If there is a real standard or consensus about math in schools, it seems to be around the value of training, not the value of education. Knowing things is something measurable and quantifiable, and it leads to societal rewards. For example, learning about batting averages or average cost is quite good, having concrete experiences which relate this "mathematics" to the real world is quite good. This knowledge is useful in life, in the workplace. Concept as tool makes lots of sense. But for a university mathematician coming into the schoolroom for the first time in many years, this "relating of mathematics to real world experience and utility" looks like training, something more like grammar as opposed to literature. It seems to have very little to do with the real mathematics of averaging, mathematics which begins in the mystery of number as symbol and reality played out in the mind of a second-grader, and which culminates in the Central Limit Theorem in probability or the heat equation in global analysis. The training part is clearly useful and desirable for most students. The case for the pursuit of what a professional mathematician thinks of as "real mathematics" in the schoolroom is, I believe, far weaker. And it is only compelling to me for two groups, those with the support structure and personal discipline to take to it when given the opportunity, and the small minority of students and teachers who cannot help but think about it. It is not a question of advanced mathematics versus elementary or beginning mathematics, but some sort of special sense of and reverence for the concepts of number and space in their own right.

It is not clear to me that we will not do just as well to accept our consensus around training, train intelligently and attractively, and make the "other" kind of mathematical experience available to teachers and students when possible, but not expect it of them. As for the "other" mathematics, perhaps we should rely on the mysterious chemistry of individual human curiosity and ego. My guess is that real data on such issues, "systemic" data and, in particular, data which does not result from love, labor and energy-intensive perturbations of "the system," is still many years away.

From A Principal's Perspective

by

Carol Lubomudrov

It was probably a good thing that at the beginning of the EMTTP project we hadn't the faintest idea of the complexity of the work we were about to undertake. We had given a lot of thought to what we hoped to accomplish and made some detailed plans. But, what we came to see as a central feature of the project, a collaboration between school and university faculty in talking about mathematics education, was not something that could be carefully laid out ahead of time. It had to evolve and that process required each of us involved to grow, change, and learn from each other. And learn we did!!!!

First of all, I was a new principal to Washington Elementary. Not only was I new, but it was my first experience at an inner city school with a large at- risk population. My previous eight years as principal had been spent in a private, college preparatory school where the very strong parental support and the competitive spirit made most students focus on learning. One of my first rude awakenings was to the fact that, in spite of the continued efforts of a dedicated faculty and a mosaic of programs, Washington Elementary School had been placed on probation by Title One because of declining scores on the Math Application and Math Concepts sections of the SAT test. I spent my first year examining every article, book, and research source I could find on at-risk education and came to the conclusion that the traditional method of drilling and timing the students on math computation was not enough to give these students the needed foundation for a strong mathematical understanding. Desperate for some answers and some help for the faculty and students, I contacted Anne Roberts and Herb Clemens at the University of Utah Math Department. We had previously worked together and had begun some very productive dialogue on teaching math in the elementary schools.

We felt that we had to go beyond simply looking at teaching techniques to a basic philosophical change in how one viewed student learning and the subject matter content. Too often schools ask "How do we teach a subject?", rather than "What is the subject about and how do students learn the subject?". This is not surprising given that the first question is often easier to answer and the fact that few elementary school teachers have had the opportunity to consider the second question in the area of mathematics. Our hope in the EMTTP project was to provide just such an opportunity.

In order to understand the breadth and complexity of this project, some discussion is needed as to the basic philosophy which drove its conceptualization and implementation. Three main questions that I felt needed consideration were: 1) what model of staff development would allow the teachers to examine mathematical concepts; 2) what experiences would encourage the faculty to play an active role in their own learning and examine changes that they might want to make in their own teaching; and 3) how could student learning about both mathematical concepts and technical skills best be facilitated. With these ideas in mind, we decided to develop a project around an intensive summer workshop

followed by year long small group discussions as well as individual classroom observations and support. Our goal was to provide opportunities for the teachers to be learners in mathematics. In order to support faculty learning, I felt that it was important for the teachers to have opportunities to view different models of teaching mathematics and to consider the mathematics that they were learning in connection with their own classroom. Consequently, the summer workshop would offer demonstration lessons that the teachers could observe and then analyze in debriefing sessions. This discussion about math and about teaching would continue during the school year with teachers examining their own lessons in grade level groups.

This was an extremely ambitious plan and no principal could "pull" the project off without a very committed faculty. That we even considered undertaking such an endeavor was due to the fact that the faculty at Washington was unique. Very early in the project development we began dialoguing with the faculty about their needs and about how the project could be designed to serve those needs. The faculty at Washington were quite diverse. Yet 80% of them were willing to commit summers for two years in order to be part of this project. They were willing to spend time and energy to become learners themselves. Interest in mathematics education is not a strength of most elementary school teachers yet this group of faculty were willing to invest of themselves in forming a partnership with the mathematic faculty at the University of Utah. We found out even more how committed the faculty were when we saw the efforts they continually made during the next two years.

We take it for granted that students should be learning in schools, but I believe that adults should be continually learning as well and facilitating that learning is one of the major roles of a principal. One basic assumption in the project was that schools are places where all members of the community should be actively engaged in learning. But a distinction needs to be made between training and learning. Training to me is done in short, one or two day classes which seem to form the backbone of teacher inservice. Teachers are shown specific techniques and, if the class is particularly valuable, they come away with a "product" to be used in the classroom. Learning takes time. It requires a dialogue among the participants. Ideas and different approaches are debated, faculty are encouraged to discuss the strengths and weaknesses of techniques, implementation is undertaken and then debriefing occurs - what worked and what didn't work. Then the process begins again.

Continuous learning for adults is important but for faculty involved in the education of young people it is critical. Teachers model both implicitly and explicitly an attitude toward learning. When teachers are actively involved in a process of learning themselves, they are more able to understand how learning occurs for their students. The active modeling of the struggle for understanding, the questioning of why and the joy of discovery allows students to engage in the essence of education.

The ideal situation, in my opinion, is to be able to engage the entire school faculty in the examination and implementation of one curriculum focus. Of course, I am talking about elementary schools where there is some commonality across grade levels. Almost all

faculty will teach math, and there is a vested interest in what other teachers at other grade levels are doing. It is important to focus on one area because I often feel that elementary school faculty are overwhelmed with all of the "subject" areas they are required to teach. Trying to incorporate all of the different federal, state and local curriculum requirements leaves them with a curriculum that becomes so fragmented that it is difficult to create a meaningful and coherent experience for the students. By focusing on one area, the faculty develops a common frame of reference, a common vocabulary and deals with common issues. This creates an excitement throughout a school as dialogue centers on the experience. Faculty meetings can focus on issues which arise concerning the joint experience and choices concerning curriculum direction can be debated. Particularly rewarding for a principal is to discover that conversations in the faculty lounge revolved around a particularly rewarding or frustrating experience that a faculty member had just had in the math classroom with most of the other faculty members interested in trying to help determine what math concepts the students were not understanding. As one faculty member put it, "Math was in the air" for the two years that we worked on this project.

The kind of collegial exchange I wanted to foster was one in which faculty members themselves initiate and sustain professional discussions. What is important in a constructive discussion around the results of a lesson is that this analysis forces us as teachers to focus on the students' growth in understanding. This moves teaching into another realm - one where the emphasis is on fostering the student's learning rather than imparting information. This is critical in math. Too often we have evaluated what students know by the brief mathematical answers that they produce rather than by the long term thinking, mathematical reasoning that brought them to that answer. It is this reasoning and thinking that is the basis for long term growth.

As you can see from the above discussion, we had lofty goals. In working toward these goals, we all experienced new roles and learning situations. The impact on me, personally, was that I was continually forced to reflect on my own philosophy of teaching, learning and especially administration. A suggestion-something along these lines: If I valued a school where learning together is the norm, then it was important for me to provide a supportive environment for this learning to take place. Within the typical elementary school structure it is not easy to carve out regular opportunities for faculty to meet. Given the many immediate problems faculty and administrators need to address in elementary schools, it is often hard to acknowledge that long-term change requires time to develop. As principal I needed to support for the faculty's efforts, their successes and their failures as they tried out new ideas, and this required my own involvement in the project activities. I met regularly with the faculty and attended as many small group discussion sessions as possible. I met with Anne Roberts systematically to discuss the progress of the project. To the extent that faculty pick up on implicit expectations of their administrators, I made every effort to express and show the value I placed on their own learning and professional development. One of the areas in which I could help as a principal was in my ability to find support for the faculty in the following areas: released time to meet, substitutes to cover classes for other classroom observations, supply and material monies, grants which gave peripheral support for materials useful to the project.

This project constantly "pushed" me to examine my philosophy, to struggle sorting out issues and to make modifications in basic assumptions as I had to narrow the gap between issues of belief and reality. Because the school had many ethnic groups, I became concerned that we didn't know enough about cultural learning differences to really understand student thinking. What was the impact on students who were at Washington Elementary for a brief period of time - could they pick up on the concepts? How could new teachers be integrated into the program? Not only did I need to make modifications in what I believed based on what I found possible or not possible, but I needed to be able to articulate the dilemmas which arose in order to discuss the project with others.

For example, one dilemma throughout the project was coming up with a concise description of what we were doing. Much of education is based on "canned" programs - Math Their Way, Box-It-Bag-It, textbooks, etc. So if one asks, what are you doing in math, one can answer, "Oh, I'm using the _____ Series." With this project, our main goal was to understand how the students were thinking about math and then bring together the materials from whatever sources we had available to promote further understanding of the concept being taught. We believed that this approach to math instruction was not only possible but that it was in the best interest of the students for a "long-term" understanding of mathematics. But the dilemma was, how to explain it and make it understandable to others outside of the project. This points to the tendency in education to want a "quick" fix and the reluctance to make the long term commitments needed to really understand some of the complex issues with which education has to deal.

I strongly believe that faculty are professionals and should be making thoughtful decisions concerning instruction in their classrooms. They need to be able to understand what their students are learning and then adapt/change their practices to best meet the needs of the students. This is certainly not a new belief, but educational practices often do not support this approach. Education abounds with forces which tend to take curriculum decisions away from teachers and forces teachers to concentrate on what they see as implementation rather than understanding. Where does the need to provide a uniform curriculum (most often interpreted as using the same basal text) mesh with adjusting the curriculum to best meet the learning of the students? What happens is that the focus often becomes the textbook and making sure that it is "covered" rather than the child or even the concept which is being taught. Many factors work against truly understanding what a child knows and then progressing to the next level of understanding. Reliance on a textbook often pushes teachers to use pre-packaged ideas, methods, and pedagogies. Trying to make curriculum "teacher proof" interferes with creating a genuine understanding of how children learn, an understanding of the concept being taught, the excitement of new learning situations for faculty themselves and an empowerment where the intellect of an individual is fostered. The more prescribed we become as a profession, the less able we are as professionals to examine each individual child and to analyze individual thinking.

The formal part of this project ended two years ago. As I look back on the long range benefits, three lingering aspects come to mind. One is that the faculty who were actively involved did change. They changed in ways that were meaningful and appropriate for

them as individuals. There is no doubt in my mind that these faculty are not the same as they were before the project began and that their teaching has changed dramatically. Secondly, the faculty have been "empowered" to continue their professional growth. They continue to be part of other projects whose focus is mathematics such as a computer network linking teachers across the country (*Teachers Resource Network*, funded by the DeWitt Wallace–Reader's Digest Fund and directed by EDC's Center for Development of Teaching). Thirdly, math has changed for the faculty and therefore for the students who are in these teachers' classrooms. We were taken off probation, student scores in math rose, and students' basic confidence in their ability to do math increased.

Bibliography

AAUW Educational Foundation. (1992). How Schools Short Change Girls. Wellesley, MA: Author.

American Mathematical Society, National Policy Statement 1994-1995, *Notices American Math. Soc.*, May/June 1994, p 437-441.

Ball, D. L. (1989). Breaking with experience in learning to teach mathematics: The role of a Preservice methods course. East Lansing, MI: National Center for Research on Teacher Education, Issue Paper 89-10.

Ball, Deborah Loewenberg (1990), 'Reflections and Deflections of Policy: The Case of Carol Turner'. *Educational Evaluation and Policy Analysis* 12(3), 247-60.

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. The Elementary School Journal, 93(4), 373-398.

Burns, Marilyn (1991). *Math by all means: Multiplication Grade 3.* New Rochelle, NY: Cuisenaire Company of America.

Cohen, David K.; Ball, Deborah Loewenberg, Policy and Practice: An Overview, *Educational Evaluation and Policy Analysis*, Fall 1990, p 233-237.

Clemens, Herb (1995), Can University Math People Contribute Significantly to Precollege Mathematics Education (Beyond Giving Future Teachers a Few Preservice Math Courses)? *CBMS Issues in Mathematics Education* Vol 5, 55-59.

Copeland, Richard W. (1970), How Children Learn Mathematics, The Macmillan Company, Collier-Macmillan Limited, London.

Corwin, R. and Russell, S.J. (1990). Used Numbers: Real data in the classroom, Palo Alto, CA: Dale Seymour.

Corwin, R., Russell, S.J., Tienery, C.C. (1990). Seeing Fractions: A Unit for the Upper Elementary Grades, California Dept. of Education, Sacramento, 1991.

Feiman-Nemser, S. and Buchmann, M. (1985). Pitfalls of experience in teacher education. Teachers College Record, 87(1), 53-65.

Gilligan, C (1982). *In a different voice.* Cambridge, MA: Harvard University Press.

Gilligan, Carol, Lyons, Nona P., Hanmer, Trudy J. (1990), Making Connections: The Relational Worlds of Adolescent Girls at Emma Willard School, Harvard University Press, Cambridge, MA.

Heaton, R.M., Lampert, M. (1993), Learning to Hear Voices: Inventing a New Pedagogy of Teacher Education. In D.K. Cohen, M.W. McLaughlin, J.E. Talbert (eds), *Teaching for Understanding* (pp 43-83). San Francisco, Jossey-Bass Publishers.

Heikkinen, H., McDevitt, T., Stone, B. (1992). Classroom Teachers as Agents of Reform in University Teacher Preparation Programs. Journal of Teacher Education, 43(4), 283-289.

Huling-Austine, L. (1990). Teacher induction programs and internships. In R. Houston (Ed.), Handbook of research on teacher education (pp. 535-549).

Jackson, Allyn, MSEB Comes of Age, *Notices American Math. Soc.*, January 1993, p 9-11.

Kamii, C. (1985). *Young children reinvent arithmetic: Implications of Piaget's Theory.* New York, Teachers College Press.

Lappan, G. Theule-Lubienski, S. (1993). Training teachers or educating professionals? What are the issues and how are they being resolved? Paper presented at ICME-7.

LeBlanc, J. , Lester, F., Kroll, D. (1992). Preparing Elementary Teachers to Teach Mathematics (PETTM). (Report No. SE 053186). Bloomington, IN: Mathematics Education Development Center. (ERIC Document Reproduction Service No. ED 349 180).
smallskip
Lester, Nancy B. and Onore, Cynthia S. (1990), Learning Change: One School District Meets Language Across the Curriculum, Boynton Cook Publishers, Heinemann, Portsmouth, New Hampshire.

National Council of Teachers of Mathematics (1989). The Curriculum and Evaluation Standards for School Mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics (1991). The Professional Standards for Teaching Mathematics. Reston, VA: NCTM.

Remillard, J. (April, 1993). Using experience to break from experience: an elementary mathematics methods course. Paper presented at the annual meeting of the American Educational Research Association, Atlanta, GA.

Remillard, Janine (1992), 'Teaching Mathematics for Understanding: A Fifth-Grade Teacher's Interpretation of Policy'. *The Elementary School Journal* 93(2), 179-93.

Roberts, A (May 1995). A Partnership in Mathematics Education: School and University Faculty Working Together, AMS Notices, 42 (5).

Roitman, Judith, Should Research Mathematicians Be Involved in K-6 Mathematics?, *Notices American Math. Soc.*, November 1992, p1062-1064.

Schifter, Deborah (1993), Reconstructing Mathematics Education: Stories of Teachers Meeting the Challenge of Reform, Teachers College, Columbia University, New York.

Schoenfeld, Alan H. (1987), Cognitive Science and Mathematics Education, Lawrence Erlbaum Associates, Publishers, Hillsdale, New Jersey.

Shavelson, R.J. and Stern, P. (1981) 'Research on Teachers' Pedagogical Thoughts, Judgements, Decisions, and Behavior'. *Review of Educational Research* 51(4), 455-98.

Thurston, William p., Mathematical Education, *Notices American Math. Soc.*, September, 1990, p 844-850.

Wilcox, S., Lanier, P., Schram, P., and Lappan, G. (1992). Influencing beginning teachers' practice in mathematics education: Confronting constraints of knowledge, beliefs, and context. Research Report 92-1. East Lansing, MI: National Center for Research on Teacher Learning, Michigan State University.